International Journal of Advanced Trends in Engineering and Technology (IJATET)
Impact Factor: 5.665, ISSN (Online): 2456 - 4664
(www.dvpublication.com) Volume 2, Issue 2, 2017

A FUZZY RELIABILITY MODEL FOR THE EFFECT OF CORTICOSTERONE BASED ON TWO PARAMETER DISTRIBUTION

P. Senthil Kumar* & Dr. A. Venkatesh**
* Assistant Professor of Mathematics, Dhanalakshmi Srinivasan College of Engineering, Perambalur, Tamilnadu
** Assistant Professor of Mathematics, A.V.V.M Sri Pushpam College, Poondi, Thanjavur, Tamilnadu

Cite This Article: P. Senthil Kumar & Dr. A. Venkatesh, "A Fuzzy Reliability Model for the Effect of Corticosterone Based on Two Parameter Distribution", International Journal of Advanced Trends in Engineering and Technology, Volume 2, Issue 2, Page Number 153-158, 2017.

Abstract:

The theoretical study for the effect of Corticosterone release scores over a 24-hours light/dark period was determined. Formulae of fuzzy two parameter Weibull distribution, Log-Logistic and Exponential distribution, fuzzy reliability function and its α-cut sets are presented. Using fuzzy reliability model based on two parameter Weibull distribution, Log-Logistic distribution and Exponential distributions, we showed that the α-cut for the fuzzy reliability for the effect of release of Corticosterone by using Weibull distribution and Exponential distribution is comparatively better than other pair.

Key Words: Fuzzy Reliability, Weibull, Log-Logistic, Exponential Distributions & Corticosterone

1. Introduction:

Many researches focused on fuzzy set theory for the fuzzy set reliability analysis. The most frequently used functions in lifetime data study are reliability or survival function. This function gives the probability of an item working for a certain amount of time without failure. Many methods and models in typical reliability theory assume that all parameters in lifetime density function are precise. But in the real world, randomness and fuzziness are mixed-up in the lifetime of the system. In 1965, Zadeh [10] introduced fuzzy set theory. Consequently, the theory and the mathematics of fuzzy sets were fleshed-out and applied in many research fields [8].The theory of fuzzy reliability was planned and developed by several authors [1], [2], [4], [5]. Cai et al [4] [5] modified the assumptions of the system is precisely defined as success or failure to fuzzy state assumption. At any time, the system may be either in the fuzzy success state or fuzzy failure state possibility assumption. The system behavior can be fully characterized by possibility measure. Cai[5] presented an introduction to system failure and its use of fuzzy methodology. In [4] [5] a method for fuzzy system reliability analysis using fuzzy number was presented. Chen S.M [2] presented a method for fuzzy system reliability analysis and alpha cuts operations of fuzzy number. Zdenek Karpisek et al [11] described fuzzy reliability function models which are based on weibull distribution. In [9] utkin et al presented a system of functional equations for fuzzy reliability analysis of various systems. In this paper we propose a fuzzy reliability model for the effect of corticosterone release in the rats over a 24-hours light/dark period based on the Weibull distribution, Log-Logistic distribution, Exponential distribution. We have determined the α-cuts of a fuzzy reliability function using two parameter weibull distribution, Log-Logistic distribution and Exponential distribution.

2. Notation:

λ	–	Scale parameter
ϕ	–	Shape parameter
t	–	Test termination time
$\overline{\lambda}[\alpha]$	–	Alpha cut of scale value
$\overline{\phi}[\alpha]$	–	Alpha cut of shape value
$\overline{R}[\alpha]$	–	Reliability function in fuzzy parameter

3. Fuzzy Reliability Model:

The Weibull distribution is usually used in statistical method analysis for life data. A continuous random variable T with two parameter Weibull distribution W (ϕ, λ) where ϕ > 0 is the shape parameter, λ > 0 is scale parameter has the probability density function

$$f(t) = \{\, \phi\lambda^{-\phi}(t)^{\phi-1}e^{-\left(\frac{t}{\lambda}\right)^{\phi}}, t \geq 0, \lambda \geq 0, \phi \geq 0\}$$

and the reliability function is

$$R_W[t] = e^{-\left(\frac{t}{\lambda}\right)^{\phi}}$$

The shape parameter gives the flexibility of Weibull distribution by changing the value of shape parameter. However sometimes we face situations when the parameter is imprecise. Therefore we consider the Weibull distribution with fuzzy parameters by replacing the scale parameter λ into the fuzzy number $\overline{\lambda}$ and shape parameter ϕ into $\overline{\phi}$. For $\alpha \in [0,1]$, the alpha cuts of fuzzy Weibull reliability function corresponding to two parameters is

$$\overline{R}_W[\alpha] = \{\overline{R}_{W1}[\alpha], \overline{R}_{W2}[\alpha]\}$$

Where $\overline{R}_{W1}[\alpha] = \inf\{e^{-\left(\frac{t}{\overline{\lambda}}\right)^{\overline{\phi}}}, \lambda \in \overline{\lambda}[\alpha], \phi \in \overline{\phi}[\alpha]\}$

$$R_{W2}[\alpha] = \inf\{e^{-\left(\frac{t}{\lambda}\right)^{\phi}}, \lambda \in \overline{\lambda}[\alpha], \phi \in \overline{\phi}[\alpha]\}$$

The Exponential distribution is expansively used in statistical method for life data. Among all statistical techniques it may be in use for engineering analysis with smaller sample sizes than any other method. A continuous random variable T with Exponential distribution $ED(\phi, \lambda)$ where, $\phi > 0$ is shape parameter and $\lambda > 0$ is scale parameter has the probability density function

$$f(t) = \lambda^{-1} e^{-\lambda(t-\phi)}, t > 0, \lambda \geq 0, \phi \geq 0$$

and the reliability function is

$$R_E[t] = e^{-\lambda(t-\phi)},$$

For $\alpha \in [0,1]$ the alpha cuts of fuzzy Exponential reliability function corresponding to two parameters is

$$\overline{R}_E[\alpha] = \{\overline{R}_{E1}[\alpha], \overline{R}_{E2}[\alpha]\}$$

Where $\overline{R}_{E1}[\alpha] = \inf\left(e^{-\overline{\lambda}(t-\overline{\phi})}, \overline{\lambda} \in \overline{\lambda}[\alpha], \overline{\phi} \in \overline{\phi}[\alpha]\right)$

$$\overline{R}_{E2}[\alpha] = \inf\left(e^{-\overline{\lambda}(t-\overline{\phi})}, \overline{\lambda} \in \overline{\lambda}[\alpha], \overline{\phi} \in \overline{\phi}[\alpha]\right)$$

The Log-Logistic distribution is extensively used in statistical method for life data. A continuous random variable T with Exponential distribution $L(\phi, \lambda)$ where, $\phi > 0$ is shape parameter and $\lambda > 0$ is scale parameter has the probability density function and the reliability function is

$$R_L[t] = \left[1 + \left(\frac{t}{\lambda}\right)^{\phi}\right]^{-1}$$

For $\alpha \in [0,1]$ the alpha cuts of fuzzy Exponential reliability function corresponding to two parameters is

$$\overline{R}_L[\alpha] = \{\overline{R}_{L1}[\alpha], \overline{R}_{L2}[\alpha]\}$$

Where $\overline{R}_{L1}[\alpha] = \inf\left[\left[1 + \left(\frac{t}{\overline{\lambda}}\right)^{\overline{\phi}}\right]^{-1}, \overline{\lambda} \in \overline{\lambda}[\alpha], \overline{\phi} \in \overline{\phi}[\alpha]\right]$

$$\overline{R}_{L2}[\alpha] = \sup\left[\left[1 + \left(\frac{t}{\overline{\lambda}}\right)^{\overline{\phi}}\right]^{-1}, \overline{\lambda} \in \overline{\lambda}[\alpha], \overline{\phi} \in \overline{\phi}[\alpha]\right]$$

4. Application:

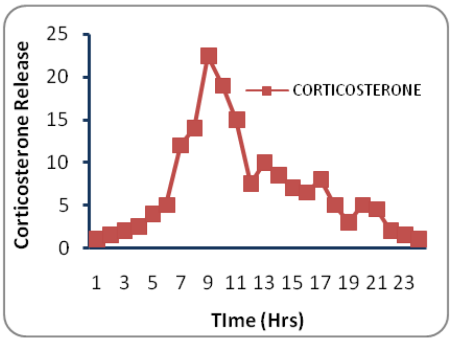

Figure 4.1: Corticosterone releases of rats over a 24- hour light/dark period

Let us consider an example for concentration of Corticosterone were determined in blood samples of rat, with free access to food and water under the condition of constant temperature and fixed 12-hours light/12-hours dark photoperiod (light on from 07.30 am to 19.30 hours) for at least two weeks prior to surgery. During this time, the rats were accustomed to the presence of the experimenter by daily handling. The experiments were carried out in early spring. The effects of Corticosterone release in rats were measured [6].

Two Parameter Weibull Distribution, scale parameter and shape parameter are

$$\lambda = 14.077 \text{ and } \phi = 1.3181$$

Let the corresponding triangular fuzzy numbers are

$$\overline{\lambda} = [14, 14.077, 15]$$

$$\overline{\phi} = [1, 1.3181, 2]$$

and the corresponding α cuts are

$$\overline{\lambda}[\alpha] = [14 + 0.077\alpha, 15 - 0.923\alpha] \text{ and } \overline{\phi}[\alpha] = [1 + 0.3181\alpha, 2 - 0.6819\alpha]$$

Two Parameter Log-Logistic Distribution, scale parameter and shape parameter are

$$\lambda = 9.4288 \text{ and } \phi = 1.7714$$

Let the corresponding triangular fuzzy numbers are

$$\overline{\lambda} = [9, 9.4288, 10]$$

$$\overline{\phi} = [1, 1.7714, 2]$$

and the corresponding α cuts are

$$\overline{\lambda}[\alpha] = [9 + 0.4288\alpha, 10 - 0.5712\alpha] \text{ and } \overline{\phi}[\alpha] = [1 + 0.7714\alpha, 2 - 0.2286\alpha]$$

Two Parameter Exponential Distribution, scale parameter and shape parameter are

$$\lambda = 1 \text{ and } \phi = 0.8696$$

Let the corresponding triangular fuzzy numbers are

$$\overline{\lambda} = [0, 1, 2]$$

$$\overline{\phi} = [0, 0.8696, 1]$$

and the corresponding α cuts are

$$\overline{\lambda}[\alpha] = [0 + \alpha, 2 - \alpha] \text{ and } \overline{\phi}[\alpha] = [0 + 0.8696\alpha, 1 - 0.1304\alpha]$$

Table 4.1: Fuzzy Reliability function based on weibull distribution

α	**λ1**	**λ2**	**Φ1**	**Φ2**	**R1(x)**	**R2(x)**
0	1	2	14	15	0.3679	1
0.1	1.031	1.931	14.008	14.908	0.521	0.9999
0.2	1.062	1.862	14.015	14.815	0.6503	0.9999
0.3	1.093	1.793	14.023	14.723	0.7502	0.9998
0.4	1.124	1.724	14.031	14.631	0.8237	0.9997
0.5	1.155	1.655	14.039	14.539	0.8761	0.9993
0.6	1.186	1.586	14.046	14.446	0.913	0.9987
0.7	1.217	1.517	14.054	14.354	0.9387	0.9975
0.8	1.248	1.448	14.062	14.262	0.9566	0.9949
0.9	1.279	1.379	14.069	14.169	0.9691	0.9895
1	1.31	1.31	14.077	14.077	0.9779	0.9779

Table 4.2: Fuzzy Reliability function based on Log-Lagistic distribution

α	**λ1**	**λ2**	**Φ1**	**Φ2**	**R1(x)**	**R2(x)**
0	1	2	9	10	0.5	0.999
0.1	1.0771	1.9771	9.0429	9.9429	0.6619	0.9989
0.2	1.1543	1.9543	9.0858	9.8858	0.7865	0.9987
0.3	1.2314	1.9314	9.1286	9.8286	0.8699	0.9985
0.4	1.3086	1.9086	9.1715	9.7715	0.9218	0.9982
0.5	1.3857	1.8857	9.2144	9.7144	0.9528	0.9979
0.6	1.4628	1.8628	9.2573	9.6573	0.9713	0.9975
0.7	1.54	1.84	9.3002	9.6002	0.9823	0.9971
0.8	1.6171	1.8171	9.343	9.543	0.9889	0.9967
0.9	1.6943	1.7943	9.3859	9.4859	0.993	0.9961
1	1.7714	1.7714	9.4288	9.4288	0.9955	0.9955

Table 4.3: Fuzzy Reliability function based on Exponential distribution

α	λ1	λ2	Φ1	Φ2	R1(x)	R2(x)
0	0	1	0	2	1	2.7183
0.1	0.0087	0.9087	0.1	1.9	0.9922	2.2656
0.2	0.0174	0.8174	0.2	1.8	0.9862	1.9231
0.3	0.0261	0.7261	0.3	1.7	0.9819	1.6624
0.4	0.0348	0.6348	0.4	1.6	0.9793	1.4636
0.5	0.0435	0.5435	0.5	1.5	0.9785	1.3123
0.6	0.0522	0.4522	0.6	1.4	0.9793	1.1983
0.7	0.0609	0.3609	0.7	1.3	0.9819	1.1143
0.8	0.0696	0.2696	0.8	1.2	0.9862	1.0554
0.9	0.0783	0.1783	0.9	1.1	0.9922	1.018
1	0.087	0.087	1	1	1	1

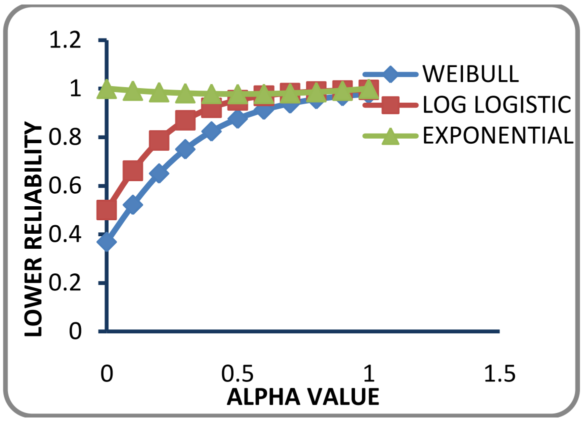

Figure 4.2: Lower alpha cut for the reliability function.

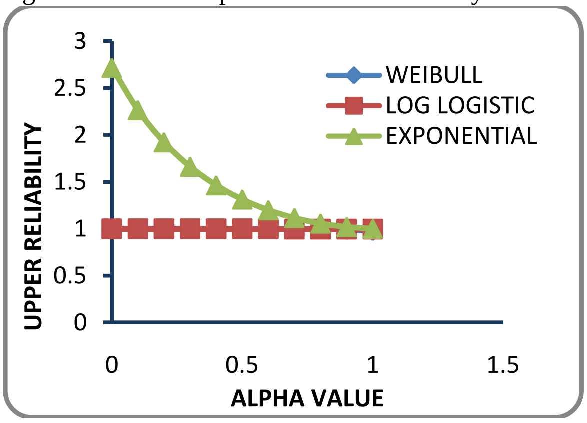

Figure 4.3: Upper alpha cut for the reliability function.

Hypothesis Testing:

Lower Fuzzy Reliability Method:

Null hypothesis H_{WL0} : There is no significant difference between the Lower Fuzzy Reliability in Weibull distribution and Log- Logistic distribution.

Alternative hypothesis $H_{WL1} : X_1 \neq X_2$

Null hypothesis H_{WE0} : There is no significant difference between the Lower Fuzzy Reliability in Weibull distribution and Exponential distribution.

Alternative hypothesis $H_{WE1} : X_1 \neq X_3$

Null hypothesis H_{LE0} : There is no significant difference between the Lower Fuzzy Reliability in Log-Logistic distribution and Exponential distribution.

Alternative hypothesis $H_{LE1} : X_2 \neq X_3$

Table 4.4: Calculation of Sample Means and Standard Deviations of Lower Reliability

α	X1	X2	X3	S1*S1	S2*S2	S3*S3
0	0.3679	0.5	1	0.1824144	0.14055001	0.0001664
0.1	0.521	0.6619	0.9922	0.075076	0.045369	0.0000260

0.2	0.6503	0.7865	0.9862	0.0209381	0.00781456	0.0000008
0.3	0.7502	0.8699	0.9819	0.002007	0.000025	0.0000270
0.4	0.8237	0.9218	0.9793	0.0008237	0.00219961	0.0000608
0.5	0.8761	0.9528	0.9785	0.0065772	0.00606841	0.0000740
0.6	0.913	0.9713	0.9793	0.013924	0.00929296	0.0000608
0.7	0.9387	0.9823	0.9819	0.0206497	0.01153476	0.0000270
0.8	0.9566	0.9889	0.9862	0.0261146	0.012996	0.0000008
0.9	0.9691	0.993	0.9922	0.0303108	0.01394761	0.0000260
1	0.9779	0.9955	1	0.0334524	0.01454436	0.0001664
TOTAL	8.7445	9.6239	10.858	0.4122879	0.26434228	0.0006362
AVG	0.795	0.8749	0.9871	0.0412288	0.026434228	0.0000636

$$\overline{X_1}=\frac{\sum X_1}{N_1},\overline{X_2}=\frac{\sum X_2}{N_2},\overline{X_3}=\frac{\sum X_3}{N_3},S_1^2=\frac{\sum(X_1-\overline{X_1})^2+(\sum X_2-\overline{X_2})^2}{N_1+N_2}$$

$$S_2^2=\frac{\sum(X_2-\overline{X_2})^2+(\sum X_3-\overline{X_3})^2}{N_2+N_3},S_3^2=\frac{\sum(X_1-\overline{X_1})^2+(\sum X_3-\overline{X_3})^2}{N_1+N_3}$$

$$\overline{X_1}=0.795,\overline{X_2}=0.8749,\overline{X_3}=0.9871,S_1^2=0.4122879,S_2^2=0.26434228,S_3^2=0.0006362$$

Calculated of $|t_{WL}|=1.019329,|t_{LE}|=2.2853,|t_{WE}|=3.1355169$

Upper Fuzzy Reliability Method:

Null hypothesis H_{WL0} :There is no significant difference between the Upper Fuzzy Reliability in Weibull distribution and Log- Logistic distribution.

Alternative hypothesis $H_{WL1}:Y_1\neq Y_2$

Null hypothesis H_{WE0} :There is no significant difference between the Upper Fuzzy Reliability in Weibull distribution and Exponential distribution.

Alternative hypothesis $H_{WE1}:Y_1\neq Y_3$

Null hypothesis H_{LE0} :There is no significant difference between the Upper Fuzzy Reliability in Log-Logistic distribution and Exponential distribution.

Alternative hypothesis $H_{LE1}:Y_2\neq Y_3$

Table 4.5: Calculation of Sample Means and Standard Deviations of Upper Reliability

α	Y1	Y2	Y3	S1*S1	S2*S2	S3*S3
0	1	0.999	2.7183	0.0091585	0.00968256	1.092443
0.1	0.9999	0.9989	2.2656	0.0091776	0.00970225	0.3510563
0.2	0.9999	0.9987	1.9231	0.0091776	0.00974169	0.0625
0.3	0.9998	0.9985	1.6624	0.0091968	0.00978121	0.0001145
0.4	0.9997	0.9982	1.4636	0.009216	0.00984064	0.0438903
0.5	0.9993	0.9979	1.3123	0.009293	0.00990025	0.1301766
0.6	0.9987	0.9975	1.1983	0.009409	0.00998001	0.225435
0.7	0.9975	0.9971	1.1143	0.0096432	0.01006009	0.3122574
0.8	0.9949	0.9967	1.0554	0.0101606	0.01014049	0.3815533
0.9	0.9895	0.9961	1.018	0.0112784	0.01026169	0.429156
1	0.9779	0.9955	1	0.0138768	0.01038361	0.4530636
TOTAL	10.957	10.974	16.731	0.1095877	0.10947449	3.4816461
AVG	1.0957	1.0974	1.6731	0.0109588	0.010947449	0.3481646

$$\overline{Y_1}=\frac{\sum Y_1}{N_1},\overline{Y_2}=\frac{\sum Y_2}{N_2},\overline{Y_3}=\frac{\sum Y_3}{N_3},S_1^2=\frac{\sum(Y_1-\overline{Y_1})^2+(\sum Y_2-\overline{Y_2})^2}{N_1+N_2}$$

$$S_2^2=\frac{\sum(Y_2-\overline{Y_2})^2+(\sum Y_3-\overline{Y_3})^2}{N_2+N_3},S_3^2=\frac{\sum(Y_1-\overline{Y_1})^2+(\sum Y_3-\overline{Y_3})^2}{N_1+N_3}$$

$$\overline{Y_1}=1.0957,\overline{Y_2}=1.0974,\overline{Y_3}=1.6731,S_1^2=0.038094,S_2^2=3.1863,S_3^2=3.1957024$$

Calculated of $|t_{WL}|=0.038094,|t_{LE}|=3.1863,|t_{WE}|=3.1957024$

Test of Hypothesis has been carried out for three distribution which are summarized as follows:

Table 4.6: Paired sample t-test for fuzzy Reliability Model for the effect of Corticosterone based on two parameter distributions

Test	**Calculated value**		**Table Value**	**Hypothesis**		**d. f**	**Result**
	Lower Fuzzy Reliability	**Upper Fuzzy Reliability**		**Lower Fuzzy Reliability**	**Upper Fuzzy Reliability**		
t_{WL}	1.019329	0.038094	2.086	Accept	Accept		There is no significant difference between the Fuzzy Reliability in Weibull distribution and Log- Logistic distribution.
t_{LE}	2.2853	3.1863	2.086	Reject	Reject	5%	There is significant difference between the Fuzzy Reliability in Log- Logistic distribution and Exponential distribution
t_{WE}	3.1355169	3.1957024	2.086	Reject	Reject		There is significant difference between the Fuzzy Reliability in Weibull distribution and Exponential distribution
t_{WL}	1.019329	0.038094	2.845	Accept	Accept		There is no significant difference between the Fuzzy Reliability in Weibull distribution and Log- Logistic distribution.
t_{LE}	2.2853	3.1863	2.845	Accept	Reject	1%	There is significant difference between the Fuzzy Reliability in Log- Logistic distribution and Exponential distribution
t_{WE}	3.1355169	3.1957024	2.845	Reject	Reject		There is significant difference between the Fuzzy Reliability in Weibull distribution and Exponential distribution

5. Conclusion:

In this paper, we have reported that the study for the fuzzy reliability for the effect of release Corticosterone for α values. Using fuzzy two parameters in Weibull distribution, Log-Logistic distribution, Exponential distribution and apply Student distribution. It is clear that the α-cut for the fuzzy reliability for the effect of release of Corticosterone by using Weibull distribution and Exponential distribution is comparatively better than other pair. We hope that this work may be used to analyze the reliability for the effect of release of Corticosterone.

6. References:

1. Chen, C.H., Mon, L., “Fuzzy System reliability analysis by interval of confidence”, fuzzy sets and systems, Vol. 56, pp. 29-36, 1993.
2. Chen. S. M., “New method for fuzzy system reliability analysis”, Cybernetics and systems, Vol. 27, pp. 385-401, 1996.
3. Dunn. J. L., Scheving and P.Millet, “Circadian variations in stress-evoked increase in plasma corticosterone”, Am J Physiol Vol. 223, pp. 402-406, 1972.
4. K.Y.Cai, C. Y. Wen and M.L. Zhang, “Fuzzy reliability modeling of gracefully degradable computing systems”, Reliability Engineering and systems safety, Vol. 33, pp.141-157, 1991.
5. K.Y.Cai, Wen and M.L. Zhang., “Fuzzy variables as a basis for a theory of fuzzy reliability in the possibility context”, Fuzzy Sets and Systems, Vol. 42, pp. 145-172, 1991.
6. Seggie, J.B. E., Shaw, I.Uhler and G.M. Brown, “Baseline 24-hour plasma corticosterone rhythm in Normal”, Sham operated and septally lesioned rats, Neuroendocrinology, Vol. 15, pp.51-61, 1974.
7. S.F., De Boer and J. Van Der Gugten, “Daily Variations in Plasma Noradrenaline, Adrenaline and Corticosterone Concentrations in Rats”, Physiology Behavior, Vol. 40, pp 323-328, 1987.
8. Singer, D., “A fuzzy set approach to fault tree and reliability analysis”, Fuzzy sets and system, Vol. 34(2), pp.145-155, 1990.
9. Utkin. L. V and Gurov. S.V, “A general formal approach for the fuzzy reliability analysis in the possibility context”, Fuzzy sets and systems, Vol. 83, pp.203-213, 1996.
10. Zadeh, L.A., “Fuzzy sets”, Information and control, Vol. 8, pp.338-352, 1965.
11. Zdenek Karpisek, Pert Stepanek, PetrJunrak, “Weibull fuzzy probability distribution for the reliability of concrete structures”, Engineering Mechanics, Vol.17, pp.363-372, 2010.