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INTUITUIONISTIC FUZZY ANTI n-NORMED LINEAR SPACE

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Abstract:

The main purpose of this paper to introduce the concept of Intuitionistic fuzzy anti n-normed linear space**.** In this paper we have introduced the definition of intuitionistic fuzzy anti n-normed linear space and gave some example on it.

Key Words: Fuzzy Set, Intuitionistic Fuzzy Set, Normed Linear Space, Fuzzy Normed Linear Space, Fuzzy N-Normed Linear Space, Fuzzy Anti N-Nomred Linear Space & Intuitionistic Fuzzy Anti n-Normed Linear Space

Introduction:

The concept of fuzzy set was introduced by Zadeh in 1965 and thereafter several authors applied it in different brances of pure and applied Mathematics. The idea of Intuitionistic fuzzy set was published by K. T. Atanassov. The concept of fuzzy norms was introduced by Katsaras in 1984. In 1992 Felbin introduced the concept of fuzzy normed linear space. The concept of Fuzzy 2-normed linear space was introduced by A. R. Meenakshi and R. Gokilavani in 2001. Thangaraj Beulla. Jebril and Samanta gave the definition of Fuzzy anti normed linear space in 2011. A. L. Narayanan and S. Vijaya Balaji have introduced the notion of Fuzzy n-normed linear space. In this paper we introduced the Intuitionistic fuzzy anti n-normed linear space.

Preliminaries:

Definition 1.1: A set is defined by a membership function or characteristic function is called a fuzzy set. A fuzzy set is defined by,

$$A=\{x,\mu_A(x)\} \text{ such that } x\in X \text{ and } \mu_A(x)\in[0,1] \text{ is called membership grade.}$$

Definition 1.2: An intuitionistic fuzzy set A is a non-empty set X is an object having the form,

$$A=\{x,\ \mu_A(x),\ \nu_A(x) \text{ where } x\in X\}$$

Where the function μ_A:E→[0,1] and ν:E→[0,1] denotes respectively the degree of membership and non-membership of the element x∈E, such that $0\le\mu_A(x)+\nu_A(x)\le 1$ for every x∈E.

Definition 1.3: Let N be a complex (or real) linear space. A norm on N is a function‖ ‖: N →R such that for all x, y ∈ N and all α ∈C(or R)

1. ‖x‖≥0 and ‖x‖≥0 0if and only if x=0‖
2. ‖x+y‖≤‖x‖+‖y‖
3. ‖αx‖=|α|‖x‖

The non-negative real number ‖x‖ is the abstract analogue of the length in Euclidean space. A linear space with a norm ‖ ‖ defined on it as above is called normed linear space.

Definition 1.4: Let X be a linear space over the field F. A fuzzy subset N of X× R(R is the set of all real numbers) is called a fuzzy norm on X if and only if for all x,u∈X and c∈K,

1. for all t∈R, with t≤0, N(x,t)=0
2. for all t∈R, with t>0, N(x,t)=1 if and only if x=0
3. for all t∈R, with t>0, N(cx,t)=N(x,t/|c|) if c≠0
4. for all s,t∈R, x,u∈X, N(x+u,s+t)≥min{N(x,s),N(x,t)}
5. N(x,∗) is non-decreasing function of R and $\lim_{t\to\infty} N(x,t) = 1$

The pair (X, N) will be referred as fuzzy normed linear space.

Definition 1.5: Let X be a linear space over the field. A fuzzy subset N of X× X ×R (R is the set of real numbers) is called a fuzzy 2-normed norm onX if and only if,

1. for all t∈R, with t≤0, $N(x_1,x_2,t) = 0$
2. for all t∈R, with t>0, $N(x_1,x_2,t) = 1$if and only if $x_1$and x_2 are linearly dependent.
3. $N(x_1,x_2,t)$ is invariant under any permutation of $x_1$and x_2.
4. for all t∈R, with t>0, $N(x_1,cx_2,t) = N(x_1,x_2,t/|c|)$ if c≠0 and c∈F
5. for all s,t∈R, $N(x_1,x_2 + x_2',s + t) \ge \min\{N(x_1,x_2,s), N(x_1,x_2',t)\}$
6. $N(x_1,x_2,*)$ is non-decreasing function of R and $\lim_{t\to\infty} N(x_1,x_2,t)=1$

Then (X, N) is called fuzzy 2-normed linear space.

Definition 1.6: Let X be a linear space over a real field F. A fuzzy subset N of X^nxR (R- set of all real numbers) is called fuzzy n-norm on X if and only if:

1. For all t∈R with t≤0, $N(x_1,x_2 \ldots, x_n,t)=0$

2. For all t∈R with t>0, $N(x_1, x_2 \ldots, x_n, t)=1$ if and only if $x_1, x_2, \ldots \ldots \ldots x_n$ are linearly depedent.
3. $N(x_1, x_{2,\ldots\ldots}, x_n, t)$ is invariant under any permutation of $x_1, x_2, \ldots \ldots \ldots x_n$
4. For all t∈R with t>o, $N(x_1, x_{2,\ldots\ldots}, cx_n, t) = N(x_1, x_{2,\ldots\ldots}, x_n, t/|c|)$ if c≠0, c∈F
5. For all s,t∈R, $N(x_1, x_{2,\ldots\ldots}, x_n + x_n', s+t) \leq \min\{N(x_1, x_{2,\ldots\ldots}, x_n, s), N(x_1, x_{2,\ldots\ldots}, x_n', t)\}$
6. $N(x_1, x_2, \ldots x_n, t)$ is a non-decreasing function of t∈R and $\lim_{t\to\infty} N(x_1, x_{2,\ldots\ldots}, x_n, t) = 1$

Then (X, N) is called a fuzzy n-normed linear space or in short f-n-NLS.

Definition 1.7: Let U be a linear space over a field F.A fuzzy subset N* of X×R such that for x,u∈X and c∈F,

1. for all t∈R, with t≤0,N*(x,t)=1
2. for all t∈R, with t≤0,N*(x,t)=0 if and only if x=0
3. for all t∈R, with t≤0,N*(cx,t)=N*(x,t/|c|) if c≠0 and c∈F
4. for all s,t∈R, N*(x+u, s+t)≤max{N*(x,s),N*(u,t)}
5. N*(x,t) is non-increasing function of t∈R and $\lim_{t\to\infty} N^*(x.t) = 0$

Then N is said to be a anti-norm on a linear space U and the pair (U,N*) is called fuzzy anti-normed linear space.

Definition 1.8: Let U be linear space over a real field. A fuzzy subset N* of U× U × R such that for all x,y,u∈U:

1. for all t∈R with t≤0, N*(x,y,t)=1
2. for all t∈R with t≤0, N*(x,y,t)=0 if and only if x,y are linearly dependent.
3. N*(x,y,t) is invariant under any permutation of x,y
4. for all t∈R with t≤0, N*(x,cy,t)=N*(x,y,t/|c|) if c≠0 and c∈F
5. for all s,t∈R with t≤0, N*(x,y+u,s+t)≤max{N*(x,y,s),N*(x,y,t)}
6. N*(x,y,t) is a non-incresing function for t∈R and $\lim_{t\to\infty} N^*(x.y,t) = 0$, Then N* is said to be a fuzzy anti2-norm on a linear space U and the pair (U,N*) is called a fuzzy anti 2-normed linear space or in short Fa-2-NLS.
   The following condition of fuzzy anti2-norm N* will be later on
7. for all t∈R with t≤0, N*(x,y,t)<1 implies that x,y are linearly dependent

Definition 1.9: Let X be a linear space over the field F. A fuzzy subset N* of X×X×….×X×R (R is the set of all real numbers) is called a fuzzy anti-norm on X if and only if,

1. for all t∈R, with t≤0, $N^*(x_1, x_2, \ldots \ldots . x_n, t)=1$
2. for all t∈R, with t>0, $N^*(x_1, x_2, \ldots \ldots . x_n, t)=0$ if and only if $x_1, x_2, \ldots \ldots . x_n$ are linearly independent.
3. $N^*(x_1, x_2, \ldots \ldots . x_n, t)$ is invariant under any permutation of $x_1, x_2, \ldots \ldots . x_n$.
4. for all t∈R, with t>0, $N^*(x_1, x_2, \ldots cx_n, t) = N^*(x_1, x_2, \ldots \ldots cx_n, t/|c|)$ where c≠0, c∈F.
5. for all s,t∈R, with t>0, $N^*(x_1, x_2, \ldots \ldots . x_n + x_n', s+t) \leq \max\{N^*(x_1, x_2, \ldots \ldots x_n, s), (x_1, x_2, \ldots \ldots x_n', t)\}$
6. $N^*(x_1, x_2, \ldots \ldots . x_n, *)$ is non-increasing function of t∈R and $\lim_{t\to\infty} N*(x_1, x_2, \ldots \ldots . x_n, t)=0$.

Then (X,N*) is called fuzzy anti n-normed linear space.

Definition 1.10: Let X be a linear space over the field F. A intuitionistic fuzzy subsets $\mu^*$of X×X×… ×X × R and v^* of X×X×… ×X × R (R is the set of all real numbers) is called a intuituionistic fuzzy anti n – norm on X if and only if

1. for all t∈R with t ≤ 0, $\mu^*(x_1, x_2, \ldots., x_n, t) = 1$ and $v^*(x_1, x_2, \ldots \ldots \ldots., x_n, t) = 1$
2. for all t∈R with t > 0 , $\mu^*(x_1, x_2, \ldots \ldots \ldots., x_n, t) = 0$ if and only if $x_1, x_2, \ldots \ldots \ldots., x_n$ are linearly independent and for all t∈R with t > 0, $v^*(x_1, x_2 \ldots., x_n, t) = 0$ if and only if $x_1, x_2, \ldots., x_n$ are linearly independent .
   $\mu^*(x_1, x_2, \ldots \ldots \ldots., x_n, t)$ and $v^*(x_1, x_2, \ldots \ldots \ldots., x_n, t)$ is invariant under any permutation of $x_1, x_2, \ldots \ldots \ldots., x_n$.
3. for all t∈R, with t > 0, $\mu^*(x_1, x_2, \ldots \ldots \ldots., cx_n, t) = (x_1, x_2, \ldots \ldots \ldots., x_n, t/|c|)$ where c ≠0 , c∈F for all t∈R, with t > 0, $v^*(x_1, x_2, \ldots \ldots \ldots., dx_n, t) = v^*(x_1, x_2, \ldots \ldots \ldots., x_n, t/|d|)$ where d≠0, d∈F.
4. for all s,t∈R, with t > 0 $\mu^*(x_1, x_2, \ldots \ldots \ldots., x_n + x_n', s+t) \leq \max\{\mu^*(x_1, x_2, \ldots \ldots \ldots., x_n, s)$ $\mu^*(x_1, x_2, \ldots \ldots \ldots., x_n', t)\}$ for all s,t∈R, with t > 0,
   $$v^*(x_1, x_2, \ldots \ldots., x_n + x_n', s+t) \leq \max\{v^*(x_1, x_2, \ldots \ldots \ldots., x_n, s)\ v^*(x_1, x_2, \ldots \ldots \ldots., x_n', t)\}$$

$\mu^*(x_1, x_2, \ldots \ldots \ldots., x_n, t)$ and $v^*(x_1, x_2, \ldots \ldots \ldots., x_n, t)$ are non increasing function of t∈R and $\lim_{n\to\infty} \mu^*(x_1, x_2, \ldots \ldots \ldots., x_n, t)=0$ and $\lim_{n\to\infty} v^*(x_1, x_2, \ldots \ldots \ldots., x_n, t) = 0$.

Then (x, μ^*, v^*) is called intuituionistic fuzzy anti n-normed linear space.

Example 1.11: Let (X, ‖•,...,•‖) be an n-normed linear space. Define,

$$\mu^*(x_1, x_2, \ldots \ldots \ldots., x_n, t) = \begin{cases} \dfrac{||x_1, x_2, \ldots\ldots\ldots, x_n||}{(t+||x_1, x_2, \ldots\ldots\ldots, x_n||.} & , \text{ when } t > 0, t\in R \\ 1 & , \text{ when } t < 0 \end{cases}$$

$$v^*(x_1, x_2, \ldots \ldots \ldots., x_n, t) = \begin{cases} \dfrac{||x_1, x_2, \ldots\ldots\ldots, x_n||}{(t+||x_1, x_2, \ldots\ldots\ldots, x_n||).} & , \text{ when } t > 0, t\in R \\ 1 & , \text{ when } t < 0 \end{cases}$$

Then (X, μ^*, ν^*) intuituionistic fuzzy anti n-normed linear space.

Proof:

1) For all t∈R, $t \le 0$, we have $\mu^*(x_1, x_2, \dots \dots \dots, x_n, t) = 1$ and for all t∈R, $t \le 0$, we have

$$\nu^*(x_1, x_2, \dots \dots \dots, x_n, t) = 1$$

2) For all t∈R, t >0, $\mu^*(x_1, x_2, \dots \dots \dots, x_n, t) = 0$ and $\nu^*(x_1, x_2, \dots \dots \dots, x_n, t) = 0$ iff $x_1, x_2, \dots \dots \dots, x_n$ are linearly independent.

Let us assume that, $\mu^*(x_1, x_2, \dots \dots \dots, x_n, t) = 0$

Given that $\mu^*(x_1, x_2, \dots \dots \dots, x_n, t) = \frac{||x_1, x_2, \dots\dots\dots, x_n||}{(t+||x_1, x_2, \dots\dots\dots, x_n||)}$

$$0 = \frac{||x_1, x_2, \dots\dots\dots, x_n||}{(t+||x_1, x_2, \dots\dots\dots, x_n||)}$$

$$||x_1, x_2, \dots \dots \dots, x_n|| = 0$$

So are $x_1, x_2, \dots \dots \dots, x_n$ are linearly independent.

Conversely let us assume that $x_1, x_2, \dots \dots \dots, x_n$, are linearly independent.

If $x_1, x_2, \dots \dots \dots, x_n$ are linearly independent then

$$||x_1, x_2, \dots \dots \dots, x_n|| = 0$$

Given that $\mu^*(x_1, x_2, \dots \dots \dots, x_n, t) = \frac{||x_1, x_2, \dots\dots\dots, x_n||}{(t+||x_1, x_2, \dots\dots\dots, x_n||)}$

$$= \frac{0}{(t+0)}$$

$$\mu^*(x_1, x_2, \dots \dots \dots, x_n, t) = 0$$

Similarly Let us assume that, $\nu^*(x_1, x_2, \dots \dots \dots, x_n, t) = 0$

$$\nu^*(x_1, x_2, \dots \dots \dots, x_n, t) = \frac{||x_1, x_2, \dots\dots\dots, x_n||}{(t+||x_1, x_2, \dots\dots\dots, x_n||)}$$

$$0 = \frac{||x_1, x_2, \dots\dots\dots, x_n||}{(t+||x_1, x_2, \dots\dots\dots, x_n||)}$$

$$|| x_1, x_2, \dots \dots \dots, x_n|| = 0$$

So are $x_1, x_2, \dots \dots \dots, x_n$ linearly independent.

Conversely let us assume that are $x_1, x_2, \dots \dots \dots, x_n$ linearly independent.

If $x_1, x_2, \dots \dots, x_n$ are linearly independent then $||x_1, x_2, \dots \dots, x_n|| = 0$.

$$\nu^*(x_1, x_2, \dots \dots \dots, x_n, t) = \frac{||x_1, x_2, \dots\dots\dots, x_n||}{(t+||x_1, x_2, \dots\dots\dots, x_n||)}$$

$$= \frac{0}{(t+0)}$$

$$\nu^*(x_1, x_2, \dots \dots \dots, x_n, t) = 0$$

3) We have to prove that $\mu^*(x_1, x_2, \dots \dots \dots, x_n, t)$ and $\nu^*(x_1, x_2, \dots \dots \dots, x_n, t)$ are invariant under any permutation of $x_1, x_2, \dots \dots \dots, x_n$

For all t∈R, t > 0,

$$\mu^*(x_1, x_2, \dots \dots \dots, x_n, t) = \frac{||x_1, x_2, \dots\dots\dots, x_{n-1}, x_n||}{(t+||x_1, x_2, \dots\dots\dots, x_{n-1}, x_n||)}$$

$$= \frac{||x_1, x_2, \dots\dots\dots, x_n, x_{n-1}||}{(t+||x_1, x_2, \dots\dots\dots, x_n, x_{n-1}||)}$$

$$= \mu^*(x_1, x_2, \dots \dots \dots, x_n, x_{n-1}, t)$$

Similarly for all t∈R, t > 0,

$$\nu^*(x_1, x_2, \dots \dots \dots, x_n, t) = \frac{||x_1, x_2, \dots\dots\dots, x_{n-1}, x_n||}{(t+||x_1, x_2, \dots\dots\dots, x_{n-1}, x_n||)}$$

$$= \frac{||x_1, x_2, \dots\dots\dots, x_n, x_{n-1}||}{t+||x_1, x_2, \dots\dots\dots, x_n, x_{n-1}||}$$

$$= \nu^*(x_1, x_2, \dots \dots \dots, x_n, x_{n-1}, t)$$

So $\mu^*(x_1, x_2, \dots \dots \dots, x_n, t)$ and $\nu^*(x_1, x_2, \dots \dots \dots, x_n, t)$ are invariant under any permutation of $x_1, x_2, \dots \dots \dots, x_n$.

4) To prove: For all t ∈ R,

$\mu^*(x_1, x_2, \dots \dots \dots, cx_n, t) = \mu^*(x_1, x_2, \dots \dots \dots, x_n, t/|c|)$ where c∈F, c≠ 0

$$\mu^*(x_1, x_2, \dots \dots \dots, cx_n, t) = \frac{||x_1, x_2, \dots\dots\dots, cx_n||}{(t+||x_1, x_2, \dots\dots\dots, cx_n||)}$$

$$= \frac{c||x_1, x_2, \dots\dots\dots, x_n||}{(t+c||x_1, x_2, \dots\dots\dots, x_n||)}$$

$$= \frac{||x_1, x_2, \dots\dots\dots, x_n||}{(\frac{t}{|c|}+||x_1, x_2, \dots\dots\dots, x_n||)}$$

$$= \mu^*(x_1, x_2, \dots \dots \dots, x_n, t/|c|)$$

For all t∈R, and d≠0, d∈F

$$\nu^*(x_1, x_2, \dots \dots \dots, dx_n, t) = \frac{||x_1, x_2, \dots\dots\dots, dx_n||}{(t+||x_1, x_2, \dots\dots\dots, dx_n||)}$$

$$= \frac{||x_1,x_2,\ldots\ldots\ldots,x_n,||}{(t+d||x_1,x_2,\ldots\ldots\ldots,x_n,||).}$$

$$= \frac{||x_1,x_2,\ldots\ldots\ldots,x_n\ ||}{(\frac{t}{|d|}+||x_1,x_2,\ldots\ldots\ldots,x_n\ ||).}$$

$$= \nu^*(x_1,x_2,\ldots\ldots\ldots..,x_n,t/|d|)$$

5) To prove: For all s, t ∈R, with t > 0,

$$\mu^*(x_1,x_2,\ldots\ldots\ldots..,x_n+x_n',s+t) \le \max\{(x_1,x_2,\ldots\ldots\ldots..,x_n,s)\ (x_1,x_2,\ldots\ldots\ldots..,x_n',t)\}$$

If a) s+t < 0
b)s=t=0
c)s+t >0, s>0, t<0 and s<0, t>0. The result is obvious
d) If s+t>0, s>0, t>0. Then

$$\mu^*(x_1,x_2,\ldots\ldots\ldots..,x_n+x_n',t) = \frac{t+||x_1,x_2,\ldots\ldots\ldots,x_n'||}{(s+t+||x_1,x_2,\ldots\ldots\ldots,x_n+x_n'||).}$$

$$\le \frac{||x_1,x_2,\ldots\ldots\ldots,x_n||+||x_1,x_2,\ldots\ldots\ldots,x_n'||}{(s+t+||x_1,x_2,\ldots\ldots\ldots,x_n||+||x_1,x_2,\ldots\ldots\ldots,x_n'||).}$$

If $\frac{||x_1,x_2,\ldots\ldots\ldots,x_n||}{(s+||x_1,x_2,\ldots\ldots\ldots,x_n'||)} \le \frac{||x_1,x_2,\ldots\ldots\ldots,x_n'||}{(t+||x_1,x_2,\ldots\ldots\ldots,x_n'||)}$ (1)

$$(t+||x_1,x_2,\ldots\ldots\ldots..,x_n'||).(||x_1,x_2,\ldots,x_n||) \le \{(s+||x_1,x_2,\ldots\ldots\ldots..,x_n||).\ (||x_1,x_2,\ldots\ldots\ldots..,x_n'||)\}$$

$(t.||x_1,x_2,\ldots\ldots\ldots..,x_n'||+||x_1,x_2,\ldots,x_n||.||x_1,x_2,\ldots\ldots\ldots..,x_n'|| \le$
$(s.||x_1,x_2,\ldots\ldots\ldots..,x_n|| +.||x_1,x_2,\ldots\ldots\ldots..,x_n||$
$t.||x_1,x_2,\ldots\ldots\ldots..,x_n'||+||x_1,x_2,\ldots,x_n||.||x_1,x_2,\ldots\ldots\ldots..,x_n'||-s.||x_1,x_2,\ldots\ldots\ldots..,x_n||-$
$||x_1,x_2,\ldots\ldots\ldots..,x_n||.||x_1,x_2,\ldots\ldots\ldots..,x_n'||\le 0$
$t.||x_1,x_2,\ldots\ldots\ldots..,x_n'|| - s.||x_1,x_2,\ldots\ldots\ldots..,x_n|| \le 0$ (2)

So $\frac{||x_1,x_2,\ldots\ldots\ldots,x_n||+||x_1,x_2,\ldots\ldots\ldots,x_n'||}{(s+t+||x_1,x_2,\ldots\ldots\ldots,x_n||+||x_1,x_2,\ldots\ldots\ldots,x_n'||)} - \frac{||x_1,x_2,\ldots\ldots\ldots,x_n'||}{(t+||x_1,x_2,\ldots\ldots\ldots,x_n'||} =$

$$\frac{t.||x_1,x_2,\ldots\ldots\ldots..,x_n|| - s.||x_1,x_2,\ldots\ldots\ldots..,x_n'||}{(s+t+||x_1,x_2,\ldots\ldots\ldots..,x_n||+||x_1,x_2,\ldots\ldots\ldots..,x_n'||).(t+||x_1,x_2,\ldots\ldots\ldots..,x_n'||)}$$

By (1) $\frac{||x_1,x_2,\ldots\ldots\ldots,x_n||+||x_1,x_2,\ldots\ldots\ldots,x_n'||}{(s+t||x_1,x_2,\ldots\ldots\ldots,x_n||+||x_1,x_2,\ldots\ldots\ldots,x_n'||)} \le \frac{||x_1,x_2,\ldots\ldots\ldots,x_n'||}{(s+||x_1,x_2,\ldots\ldots\ldots,x_n'||)}$

$\mu^*(x_1,x_2,\ldots,x_n+x_n',s+t) \le \mu^*(x_1,x_2,\ldots,x_n,s)$ (3)

Similarly if $\frac{||x_1,x_2,\ldots\ldots\ldots,x_n||}{(t+||x_1,x_2,\ldots\ldots\ldots,x_n'||)} \le \frac{||x_1,x_2,\ldots\ldots\ldots,x_n||}{(s+||x_1,x_2,\ldots\ldots\ldots,x_n||).}$ (4)

$\mu^*(x_1,x_2,\ldots\ldots\ldots..,x_n+x_n',s+t) \le \mu^*(x_1,x_2,\ldots\ldots\ldots..,x_n',t)$ (5)

From (3) and (5) we have,

$$\mu^*(x_1,x_2,\ldots\ldots\ldots..,x_n+x_n',s+t) \le \max\{(x_1,x_2,\ldots\ldots\ldots..,x_n,s)\ (x_1,x_2,\ldots\ldots\ldots..,x_n',t)\}$$

Similarly we get,

$$\nu^*(x_1,x_2,\ldots\ldots\ldots..,x_n+x_n',s+t) \le \max\{(x_1,x_2,\ldots\ldots\ldots..,x_n,s)\ (x_1,x_2,\ldots\ldots\ldots..,x_n',t)\}$$

6) For all t_1,t_2∈R , t_1,t_2>o, $\mu^*(x_1,x_2,\ldots\ldots\ldots..,x_n,, t)$ and $\nu^*(x_1,x_2,\ldots\ldots\ldots..,x_n,t)$ are non increasing function

If $t_1,t_2 < 0$ *then* $\mu^*(x_1,x_2,\ldots\ldots\ldots..,x_n,t_1) = \mu^*(x_1,x_2,\ldots\ldots\ldots..,x_n,t_2)$

Suppose t_1,t_2>0

$$\frac{||x_1,x_2,\ldots\ldots\ldots,x_n||}{(t_1+||x_1,x_2,\ldots\ldots\ldots,x_n||).} - \frac{||x_1,x_2,\ldots\ldots\ldots,x_n||}{(t_2+||x_1,x_2,\ldots\ldots\ldots,x_n||).} \ge 0$$

On simplification we have $\frac{(t_2-t_1)||x_1,x_2,\ldots\ldots\ldots,x_n||}{(t_1+||x_1,x_2,\ldots\ldots\ldots,x_n||)(t_2+||x_1,x_2,\ldots\ldots\ldots,x_n||).} \ge 0$

$$\frac{||x_1,x_2,\ldots\ldots\ldots,x_n||}{(t_1+||x_1,x_2,\ldots\ldots\ldots,x_n||).} \ge \frac{||x_1,x_2,\ldots\ldots\ldots,x_n||}{(t_2+||x_1,x_2,\ldots\ldots\ldots,x_n||).}$$

$$\mu^*(x_1,x_2,\ldots\ldots\ldots..,x_n,t_1) \ge \mu^*(x_1,x_2,\ldots\ldots\ldots..,x_n,t_2)$$

So $\mu^*(x_1,x_2,\ldots\ldots\ldots..,x_n,, t)$ is non increasing function.
Similarly $\nu^*(x_1,x_2,\ldots\ldots\ldots..,x_n,, t)$ is also non increasing function.

$$\lim_{t\to\infty} \mu^*(x_1,x_2,\ldots\ldots\ldots..,x_n,t) = \lim_{t\to\infty} \frac{||x_1,x_2,\ldots\ldots\ldots,x_n||}{(t+||x_1,x_2,\ldots\ldots\ldots,x_n||).} = 0$$

Thus (X, μ^*,ν^*) is an intuitionistic fuzzy anti n-normed linear space.

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