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COMPLEX ANNULUS SUB ALGEBRA'S OVER IDEALS

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Abstract:

In this paper, we introduce the complex annulus fuzzy ideals and complex annulus fuzzy sub algebra's concepts of annulus fuzzy algebra's and prove some results. Further, we investigated the domain of the complex annulus fuzzy homomorphism over sub algebra and ideals.

Key Words: Fuzzy Set, Annulus, Complex Annulus Fuzzy Ideal, Annulus Sub Algebra & Homomorphism

1. Introduction:

The concept of a fuzzy subset of a set was first considered by Zadeh [21] in 1965. Thefuzzy set theories developed by Zadeh and others have found many applications in the domain of mathematics and elsewhere. After the introduction of the concept of fuzzy sets by Zadeh [21], several researches were conducted on the generalizations of the notion of fuzzy set and application to many logical algebras such as: In 2001, Lele, Wu, Weke, Mamadou and Njock [14] studied fuzzy ideals and weak ideals in BCK-algebras. They were introduced by Imai and Iseki [6], [7] in 1966 and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. A fuzzy subset f of a set S is a function from S to a closed interval [0, 1]. In 2002, Jun, Roh and Kim [7] studied fuzzy Balgebras in B-algebras. Yonglin and Xiaohong [20] introduced the notion of fuzzy a-ideals in BCI-algebras, and investigated its properties. In 2004, Jun [9] introduced the concept of (α, β)-fuzzy ideals of BCK/BCI-algebras. In 2005, Akram and Dar [1] introduced the notions of T-fuzzy subalgebras and T-fuzzy H-ideals in BCI-algebras and investigated some of their properties. Jun [11] introduced the notion of (α, β)-fuzzy subalgebras of BCK/BCI-algebras. Akram and Dar [2] introduced the notion of fuzzy ideals in K-algebras. In 2008, Saeid and Jun [31] introduced the concept of anti fuzzy subalgebras of BCK/BCI-algebras by using the notion of anti fuzzy points. Saeid [18] introduced the notion of fuzzy dot BCK subalgebras. Karamdin [16] characterized anti fuzzy ideals in weak BCC-algebras. Jun [8] introduced the notion of Q-fuzzy subalgebras of BCK/BCI-algebras, and provided some appropriate examples. Among many algebraic structures, algebras of logic form important class of algebras. Examples of these are BCK-algebras [6], BCI-algebras [7], BCH-algebras [7], KU-algebras [16], SU-algebras [10] and others. They are strongly connected with logic. For example, BCI-algebras introduced by Iseki [5] in 1966 have connections with BCI-logic being the BCI-system in combinatory logic which has application in the language of functional programming. BCK and BCI-algebras are two classes of logical algebras.In 2011, Mostafa, Abd-Elnaby and Yousef [14] introduced the notion of fuzzy KUideals in KU-algebras. Mostafa, Omar and Marie [15] introduced the notion of anti-fuzzy sub-implicative ideal of BCI-algebras.Yamini and Kailasavalli [19] introduced the notion of B-ideals and fuzzy B-ideals in B-algebras. Rajam and Chandramouleeswaran [17] introduced the notion of L-fuzzy β-subalgebras on β-algebras, and investigated some of their properties. Al-Shehri [3] introduced the notion of anti fuzzy implicative ideals of BCK-algebras.

2. Preliminaries:

Before we begin our study, we will introduce the definition of an annulus -algebra.

Definition 2.1: An algebra B = (B , • , 0) of type (2,0) is called an annulus –algebra if it satisfies the following conditions; for any x,y,z ε B

$$(AA\text{-}1)\ (y\bullet z) . ((x \bullet y) \bullet (x\bullet z)) = 0$$
$$(AA\text{-}2)\ x\bullet 0 = 0$$
$$(AA\text{-}3)\ 0\bullet x = x \text{ and}$$
$$(AA\text{-}4)\ x\bullet y = y \bullet x = 0 \text{ implies } x =y.$$

Example 2.2: Let B = {0, 1, 2, 3} be a set with a binary operations•defined by the following table

•	0	1	2	3
0	0	1	2	3
1	0	0	0	0
2	0	1	0	3
3	0	1	2	0

Then (B , • , 0) is an annulus-algebra.

Definition 2.3: A non-empty A of B is called annulus-ideal of B if it satisfies the following properties (AI-1) the constant 0 of B is in A , and (AI-2) for any x,y,z ε B, x •(y • z) ε A and y ε A imply x • z ε A. Clearly, B and {0} are annulus-ideals of B.

Theorem 2.4: Let B be an annulus-algebra and {Ai} iεI is a family of annulus –ideals of B .Then ∩ Ai is annulus –ideal of A.

Definition 2.5: A subset S of B is called an annulus- sub algebra of B if the constant of B is in S, anf (S, • , 0) itself forms an annulus-algebra. Clearly B and {0} are annulus-sub algebra's of A.

Theorem 2.6: Let B be annulus-algebra and {Ai} iεI a family of annulus-sub algebra's of B. Then ∩Ai is an annulus-sub algebra's of B.

Proposition: A non-empty subset S of an annulus-algebra A = (A, • , 0) is an annular-algebra of B if and only if S is closed and the • multiplication of B.

Definition 2.7:[Fuzzy set]: A fuzzy set in a non-empty set X is an arbitrary function δ : X → [0,1] where [0,1] is the unit segment of the real line.

Definition 2.8: A complex fuzzy subset A, defined on a universe of discourse X, is characterized by a membership function $\tau_A(\mathrm{x})$ that assigns any element x∈X a complex valued grade of membership in A. The values of $\tau_A(\mathrm{x})$ all lie within the unit circle in the complex plane and thus all of the form $P_A(\mathrm{x})\ e^{j\mu_A(\mathrm{x})}$ where $P_A(\mathrm{x})$ and $e^{j\mu_A(\mathrm{x})}$ are both real valued and $P_A(\mathrm{x}) \in [0,1]$. Here $P_A(\mathrm{x})$ is termed as amplitude term and $e^{j\mu_A(\mathrm{x})}$ is termed as phase term.

The complex fuzzy set may be represented in the set form as A = {(x,τ_A(x)) / x∈X }. It is denoted by CFS.

The phase term of complex membership function belongs to (0,2π). Now we take those forms which Ramot.et.al presented in [8] to define the game of winner, neutral and lose.

$$\mu_{A\cup B}(\mathrm{x}) = \begin{cases} \mu_A(\mathrm{x}) & \mathrm{if} p_A > p_B \\ \mu_B(\mathrm{x}) & \mathrm{if} p_A < p_B \end{cases}.$$

This is a novel concept and it is the generalization of the concept "winner take all" introduced by Ramot.et.al [2003] for the union of phase terms.

Example 2.9: Let X = {x_1, x_2, x_3} be a universe of discourse. Let A and B be complex fuzzy sets in X as shown below.

$$A = \{0.6\ e^{i(0.8)},\ 0.3\ e^{i\frac{3\pi}{4}},0.5\ e^{i(0.3)}\}$$
$$B = \{0.8\ e^{i(0.9)},\ 0.1\ e^{i\frac{\pi}{4}},0.4\ e^{i(0.5)}\}$$
$$A\cup B = \{0.8\ e^{i(0.9)},\ 0.3\ e^{i\frac{3\pi}{4}},0.5\ e^{i(0.3)}\}$$

We can easily calculate the phase terms $e^{i\mu_{A\cap B}(\mathrm{x})}$ on the same line by winner, neutral and loser game.

Definition 2.10: A fuzzy set δ in B is called complex annulus-ideal of B if it satisfies the following properties; for any x, y, z ε B,

(i) δ (0) ≥ δ (x) and
(ii) δ(x•z) ≥ min { δ(x • (y•z)), δ (y)}.

Definition 2.11: A fuzzy set δ in B is called an complex fuzzy annulus sub algebra of B if for any x,y ε B, (x • y) ≥ min { δ(x) , δ(y) }.

Example: Let B = { 0 , a } be a set with a binary operations • defined by the following

•	0	a
0	0	a
a	0	0

Then (B , • , 0) is complex fuzzy annulus-algebra. We define a fuzzy set δ in B as follows δ (0) = 0.4 and δ(1) = 0.6. Using tis data, we can show that δ is a complex annulus fuzzy ideal of B and complex annulus fuzzy sub algebra of B.

Example 2.12: Let B = { 0 ,1,2 } be a set with a binary operations •defined by the following table

•	0	1	2
0	0	1	2
1	0	0	1
2	0	0	1

Then (B , • , 0) is complex fuzzy annulus- sub algebra. We define a fuzzy set δ in B as follows δ (0) = 0.7 and δ(1) = 0.4, δ (2) = 0.9.

Lemma 2.13: Let be a complex fuzzy set in B. Then the following statements old; for any x,y ε B,

(i) max { δ(x), δ(y) }= min { 1-δ(x) , 1-δ(y) }
(ii) 1- min { δ(x), δ(y) }= max{ 1-δ(x) , 1-δ(y) }.

3. Main Results:

Theorem 3.1: Every complex annulus fuzzy ideals of B is a complex annulus fuzzy sub algebra.

Proof: Let δ be complex annulus fuzzy ideas of B.

Let x, y ε B.

$$\delta(x, y) \geq \min\{\ \delta(x\ .\ (y\ .y\ )),\ \delta(y)\} = \min\{\delta(x\ .0), \delta(y)\} = \min\{\ \delta(0), \delta(y)\}$$
$$= \delta(y) \geq \min\{\ \delta(x), \delta(y)\}.$$

Hence δ is complex annulus fuzzy sub algebra.

Lemma 3.2: Let δ be a complex annulus fuzzy ideal of B. If the inequality $x \le y$ in B for all x, y, z ε B, then $\delta(z) \ge \min \{ \delta(x), \delta(y) \}$ for all x, y, z ε B.

Proof: Assume that $x \le y.z$ for all x, y, z ε B.

Then $x.(y.z) = 0$

By definition, we have

$$\delta(x.z) \ge \min \{ \delta(x.(y.z)), \delta(y)\} \quad \text{------------------------------} \quad (1)$$

let x=0, so

$$\delta(z) = \delta(0.z) \ge \min \{ \delta(0.(x.z)), \delta(x)\} = \min \{ \delta(x.z), \delta(x)\} \quad \text{----------------------------} \quad (2)$$

$$\delta(x.z) \ge \min \{ \delta(x.(y.z)), \delta(y)\} = \min \{ \delta(x), \delta(y)\} \quad \text{---------------------------------------} \quad (3)$$

By (2) and (3), we have

$$\delta(z) \ge \min \{\delta(x.z), \delta(y)\} \ge \min \{ \delta(x), \delta(y)\} = \min \{ \delta(x), \delta(y)\}$$

Lemma 3.3: Let δ be a complex annulus fuzzy ideal of B and if x,y ε B in such that $x \le y$ in B, $\delta(x) \le \delta(y)$ } .

Proof: Let x,y ε B be such that $x \le y$ in B. Then $x.y = 0$

Thus $\delta(y) = \delta(0.y) \ge \min \{ \delta(0.(x,y)), \delta(y)\} = \min \{ \delta(0.0), \delta(x)\} = \min \{ \delta(0), \delta(x)\} = \delta(x)$.

Lemma 3.4: Let δ is complex annulus fuzzy set in B. For any λ ε [0,1] the following properties hold;

(i) $L(\delta; \lambda) = U(\Delta; 1-\lambda)$
(ii) $L^-(\delta; \lambda) = U^+(\Delta; 1-\lambda)$
(iii) $U(\delta; \lambda) = L(\Delta; 1-\lambda)$
(iv) $U^+(\delta; \lambda) = L^-(\Delta; 1-\lambda)$.

Lemma 3.5: For any ℓ ,m ε R such that $\ell \le m$, $\ell < m+\ell/2 < m$.

Theorem 3.6: If δ is complex annulus fuzzy ideal of B, then $\delta(x, (x.y)) \ge \delta(y)$ for all x,y ε B.

Proof: Let x, y ε B. Then

$$\delta(x.(x.y)) \ge \min \{ \delta(x, ((x.y).(x.y)), \delta(x.y)\} = \min \{\delta(x.0), \delta(x.y)\} = \min \{ \delta(0), \delta(x.y)\} = \delta(x.y)$$
$$\ge \min \{ \delta(x.(y.y)), \delta(y)\} = \min \{\delta(x.0), \delta(y)\} = \min \{ \delta(0), \delta(y)\} = \delta(y).$$

Theorem 3.7: Let $(A.\bullet, 0_A)$ and $(B, *, 0_B)$ be complex annulus fuzzy algebra's and let $f: A \rightarrow B$ be an annulus homeomorphisms. Then the following statement holds;

(i) For every complex annulus fuzzy ideal of β of B , δ is complex annulus ideal of A and
(ii) For every complex annulus fuzzy sub algebra of β of B , δ is complex annulus fuzzy sub algebra of A.

Proof: (i) Let β be a complex annulus fuzzy ideal of B. Let x ε A. Then

$$\Gamma(0B) = (\beta \bullet \delta)(0_B) = \beta(\delta(0_B)) \ge \beta(\delta(x)) = (\beta.\delta)(x) = \Gamma(x).$$

Let x,y,z ε A. Then $\Gamma(x.z) = (\beta.\delta)(x.z) = \beta(\delta(x) * \delta(z)) \ge \min \{ \beta(\delta(x) *(\delta(y) * \delta(z)), \beta\delta(y)\}$

$$= \min \{\beta(\delta(x.(y.z), \beta\delta(y)\} = \min \{ (\beta \bullet \delta)(x.(y.z)), (\beta \bullet \delta)(y)\}$$
$$= \min \{ \Gamma(x.(y.z), \Gamma(y)\}.$$

Hence Γ is complex annulus fuzzy ideals of B.

(ii) Let β be the complex annulus fuzzy sub algebra of B. Let x,y ε A. Then

$$\Gamma(x.y) = (\beta \bullet \delta)(x.y) = \beta(\delta(x.y)) = \beta(\delta(x) * \delta(y))$$
$$\ge \min \{ \beta(\delta(x)), \beta(\delta(y))\} = \min \{ (\beta \bullet \delta)(x), (\beta \bullet \delta)(y)\}$$
$$= \min \{ \Gamma(x), \Gamma(y)\}.$$

Hence Γ is complex annulus fuzzy sub algebra's of B.

4. Conclusion:

We introduce the complex annulus fuzzy ideals and complex annulus fuzzy sub algebra's concepts of annulus fuzzy algebra's and prove some results. Further, we investigated the domain of the complex annulus fuzzy homomorphism over sub algebra and ideals.

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