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ON γ- CONNECTEDNESS IN L-TOPOLOGICAL SPACES

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Abstract:

In this paper, a certain new connectedness of L-fuzzy subsets in L-topological spaces is introduce and study by means of γ- closed sets. It preserves some fundamental properties of connected set in general topology. Especially the famous K. Fan's Theorem holds.

1. Introduction:

Connectivity is one of the important notions in general topology, There have been many works about connectedness in L-topological spaces (see [1, 2, 3, 6, 8, 9, 10, 11, 12, 13, 14 etc]. In this paper, we introduce a certain new connectedness in L-topological spaces by means of γ-closed sets, which is called γ –connectedness. The γ – connectedness preserves some fundamental properties of connected sets in general topology. Especially the famous K-Fan's Theorem holds for γ-connectedness.

2. Preliminaries:

Throughout this proper, (L, $\vee$, $\wedge$') is a complete De Morgan algebra, X a nonempty set .L^x the set of all L –fuzzy sets on X. The smallest element and the largest element in L^x are denoted by $\underline{0}$ and $\underline{1}$. An element a is L is called a prime element if $a\geq b\wedge c$ implies that $b\leq a$ or $a\geq c$. An element a in L is called a co-prime element if a' is a prime element [5]. The set of all nonzero co-prime elements in L is denoted y M(L). The set of all nonzero co-prime elements in L^X is denoted by $M(L^X)$. For an L-fuzzy set D, M(D) denotes the set of all nonzero co-prime elements contained in D.An L-topological space is a pair (X,τ), where τ is a subfamily of L^x which contains $\underline{0},\underline{1}$ and is closed for any suprema and finite infima. τ is called an L-topology on X. Each member of τ is called an open L-set and its quails complementation is called a closed L-set. An L'-topological space is an ordered pair (X,τ) where τ is a subfamily of L^X which contains 0,1 and is closed for any suprema and finite infima.

Definition 2.1: [7] Let (X,τ) be an L-topological space, $A\in L^X$ Then A is called an γ–open set if $A\leq Int(Cl(A)) V Cl(Int(A))$. The complement of an γ–open set is called an γ–closed set. Also, $\gamma O(L^X)$ and $\gamma C(L^X)$ will always deonote the family of all γ–open sets and γ–closed sets respectively. Obviously, $A\in\gamma O(L^X)$if, and only if $A'\in\gamma O(L^X)$.

Definition 2.2: [7] Let (L^X, τ) be an L-topological space, $A,B\in L^X$. Let $\gamma Int(A)=v\{\beta\in L^X \backslash \beta\leq A, B\in\gamma O(L^X)\}$, $\gamma Cl(A)=\wedge\{\beta\in L^X \backslash A\leq B, B\in\gamma C(L^X)\}$. Then $\gamma Int(A)$ and $\gamma Cl(A)$ are called the γ -interior and γ–closure of A respectively.

Definition 2.3: Two L-fuzzy sets A,B of an L-topological space(X,τ) are called γ -separated if $\gamma Cl(A)\wedge(B)=A\wedge\gamma Cl(B)=\underline{0}.$

Definition 2.4: In an L-topological space (X,τ), an L-fuzzy set D is called γ - connected if D cannot be represented as a union of two γ - separated nonnull sets.

Definition 2.5: For two L-topological spaces (X,τ) and (Y,σ), a mapping $f:X\rightarrow Y$ is called an γ-irresolute mapping if $f^{\leftarrow}(B)$ is γ-closed in (X,τ) for each γ–closed set B in (Y, σ).

Lemma 2.6: [12] Let $A,B\in L^X$ and $A\not\leq B$. If $1\in M(L)$,then $A'VB\neq\underline{1}$.

Lemma 2.7: [3] In a L-topological space (X,τ) , an L-fuzzy set D is called connected if there does not exist closed sets A, B such that DA, DB, $D'\not\leq A, D\not\leq B$, $D'VAVB=\underline{0}.D\wedge A\wedge B=\underline{0}.$

3. γ–Connectedness of L-Fuzzy Sets:

Definition 3.1: Let (X,τ) be an L-topological space, $D\in L^x$.D is called γ-connected if there does not exist γ-closed sets A,B such that $D'\not\leq A, D\not\leq B$, $D'VAVB=\underline{1}.D\wedge A\wedge B=\underline{0}.$

Example 3.2: .Let $X_1\cap X_2=\emptyset$, $X=X_1\cup X_2$, L=[0,1]. Defined fuzzy set $(C_\alpha,C_b)\in[0,1]^x$ as follows:

$$(C_\alpha,C_b)(x)=\begin{cases} a & \text{if } x\in X_1; \\ a & \text{if } x\in X_2; \end{cases}$$

Take $\beta=\{(C_{0.4},C_1), (C_1,C_{0.4}), (C_{0.5},C_0), (C_{0.7},C_0), (C_0,C_{0.7})\}$

Let δ be a [0,1] –topology generated by βon X. Now we prove that $(C_{0.5},C_0)$m is γ- connected. In fact, suppose that $(C_{0.5},C_{0.5})$ is γ–connected. In fact, suppose that $(C_{0.5},C_{0.5})$ is not γ- connected. Then there exist two γ- closed sets A,B such that $(C_{0.5},C_{0.5})\not\leq A$, $(C_{0.5},C_{0.5})\not\leq B$, $(C_{0.5},C_{0.5})'vAvB=\underline{1}$, $(C_{0.5},C_{0.5})\wedge A\wedge B=\underline{0}$. This implies that

$(C_{0.5},C_{0.5})\not\leq A$, $(C_{0.5},C_{0.5})\not\leq B=\underline{1}$, $A\wedge B=\underline{0}$. Obviously A(or B) satisfying $A\wedge B=\underline{0}$, must be in (C_α,C_0), $(C_0,C_\alpha)|\ \alpha\in[0.6, 0.7]\}$.But any two γ-closed sets in $\{(C_\alpha,C_0), (C_0,C_\alpha)|\ \alpha\in[0.6, 0.7]\}$ does not satisfy $AVB=\underline{1}$. Therefore $(C_{0.5},C_{0.5})$ is γ –connected.

Theorem 3.3: Let (X,τ) be an L-topological space, $D\in L^x$. If $1\in M(L)$ then the following conditions are equivalent.

1. D is γ – connected.
2. There does not exist γ–closed sets A, B such that $D\wedge A\neq 0$, $D\wedge B\neq 0$, $D'VAVB=\underline{1}$.$D\wedge A\wedge B=\underline{0}$.
3. There does not exist γ–open sets, U, V such that $D\not\leq U$, $D\not\leq V$ $D'vUvU=\underline{1}$, $D\wedge U\wedge V=\underline{0}$.
4. There does not exist γ–open sets, U, V such that $D\wedge U\neq 0$, $D\wedge V\neq \underline{0}$, $D'VUVV=\underline{1}$. $D\wedge U\wedge B=\underline{0}$.

Proof: (1) ⇒(2) . Suppose that there exist γ–closed sets A and B such that $D\wedge A\neq 0$, $D\wedge B\neq 0$, $D'VAVB=\underline{1}$. $D\wedge A\wedge B=\underline{0}$. Then obviously we have that $D'VAVB=\underline{1}$. $D\wedge A\wedge B=\underline{0}$, In this case, we can prove that $D\not\leq A$, $D\not\leq B$. In fact, if D≤A, then $D\wedge A\wedge B = D\wedge B=\underline{0}$.

(2) ⇒ (3) .Suppose that there exits γ-open sets U and V such that $D\not\leq U$, $D\not\leq V$, $D'vUvV=\underline{1}$, $D\wedge U\wedge V=\underline{0}$. Put A=U', B=V' Obviously we have $D'vAvB=\underline{1}$, $D\wedge A\wedge B=\underline{0}$. In this case, we can prove that $D\wedge A\neq\underline{0}$, $D\wedge B\neq\underline{0}$ .In fact, if $D\wedge A=\underline{0}$, then $D'vU=D'vA'=\underline{1}$, hence by Lemma 2.6 we obtain that D≤U, which contradicts $D\not\leq U$.

(3) ⇒ (4) is analogous to (1) ⇒ (2)

(4) ⇒ (1) is analogous to (2) ⇒ (3).

Theorem 3.4: Let D be an γ-connected set in (X,τ). If D≤E≤γCl(D), then E is also an γ-connected set.

Proof: Suppose that E is not γ–connected. Then there exist two γ-closed sets A and B such that $E\not\leq A$,$E\not\leq B$, $E'VAVB=\underline{1}$. $E\wedge A\wedge B=\underline{0}$. Hence $D'vAvB=\underline{0}$, $D\wedge A\wedge B=\underline{0}$. In fact, we also have that $D\not\leq A$,$D\not\leq B$ (If $D\not\leq A$, then $E\not\leq\gamma Cl(D)\leq A$, which contradicts $E\not\leq A$).This shows that D is not γ-connected in (X,τ), which contradicts that D is γ- connected. Therefore E is γ-connected.

Lemma 3.5: Let (X,τ) be an L-topological space and $D,E\in L^X$. Then D,E are γ-separated if, and only if there exists two γ- closed set A,V such that D≤A,E≤B and $(DVE)\wedge A\wedge B=\underline{0}$.

Proof: If there exist two γ-closed sets A,B such that D≤A,E≤B and $(DVE)\wedge A\wedge B=\underline{0}$, then we have that $(D\wedge\gamma Cl(E))V(\gamma Cl(D)\wedge E)\leq(D\wedge B)v(E\wedge A)=\underline{0}$. This shows that D,E are γ–separated. Conversely, if D, E and γ–separated, then $(D\wedge\gamma Cl(E))V(\gamma Cl(D)\wedge E)=\underline{0}$. Take A=γCl(D) and B=γCl(E). Then $(DVE)\wedge A\wedge B=(DVE)\wedge(\gamma Cl(D)\wedge\gamma Cl(E)= D\wedge\gamma Cl(E))V\gamma Cl(D)\wedge E)=\underline{0}$.

Theorem 3.6: Let D and E be two γ- connected L-fuzzy sets in an L-topological space (X,τ). If D and E are γ-separated, then DVE is γ-connected.

Proof: Suppose that DVE is not γ –connected. Then there exist two γ- closed sets A and B such that $DVE\not\leq A$, $DVE\not\leq B$. $(DVE)'VAVB=\underline{1}$. $(DVE)\wedge A\wedge B=\underline{0}$. Hence we have that $D'VAVB=\underline{1}$, $D\wedge A\wedge B=0$, $E'vAvB=\underline{1}$. $E\wedge A\wedge B=\underline{0}$. By $DVE\not\leq A$. We have that $D\not\leq A$ or $E\not\leq A$. Suppose that $D\not\leq A$. Then we must have that $D\not\leq B$ Since D is γ-connected. Further by $DVE\not\leq B$ we obtain that $E\not\leq B$. In this case it follows that $\not\leq A$. Therefore by $(DVE)\wedge A\wedge B=\underline{0}$ and Lemma 3.5 we know that D,E are γ–separated, which contradicts that D,E are not γ–separated. The proof is completed.

Theorem 3.7: Let $\{Dt\backslash t\in\Omega\}$ be a family of γ-connected L-fuzzy sets. If $\bigwedge_{t\in\Omega} D_t\neq\underline{0}$, then $\bigvee_{t\in\Omega} D_t$ is γ–connected.

Theorem 3.9: Let f:(X,ι)→(Y,σ) be an γ-irresolute mapping. If D is γ-connected in (X,ι), then so is f(D) in (Y, ι).

Proof: Suppose that f(D) is not γ-connected in (Y,σ). Then there exit two γ-closed sets A,B in (Y,σ) such that $f(D)\not\leq A$,$f(D)\not\leq B$, $f(D)'VAVB=\underline{1}$, $f(D)\wedge A\wedge B=\underline{0}$. Thus by $D\leq f^{\leftarrow}(f(D))$ or $D'\leq f^{\leftarrow}(f(D)')$ we have that $D\not\leq f^{\leftarrow}(A)$, $D\not\leq f^{\leftarrow}(B)$, $D'V f^{\leftarrow}(A)V$, $f^{\leftarrow}(B)=\underline{1}$, $D\wedge f^{\leftarrow}(A)\wedge D\not\leq f^{\leftarrow}(B),=\underline{0}$. Since f is γ –irresolute mapping, $f^{\leftarrow}(A)$ and $f^{\leftarrow}(B)$ are γ-closed set in (X,ι). This slows that D is not γ-connected in (X,ι) which contradicts that D is γ–connected. Therefore f(D) is γ–connected in (Y,σ).

Theorem 3.10: Let (X,ι) be an L-topological space and $D\in L^X$. Then D is -γ-connected if, and only if for any tow co-prime elements a,b≤ D, there exits and γ-connected set E such that a,b ≤E≤D.

Proof: The necessity is obvious. Now we prove the sufficiency, suppose that D is not γ-connected in (X,ι). Then there exist two γ-closed sets A,B in (X,ι) such that $D\not\leq A$, $D\not\leq B$, $D'vAvB=\underline{1}$, $D\wedge A\wedge B=\underline{0}$. Take two coprime elements a,b≤D such that $a\not\leq A$ and $b\not\leq B$. Let E be an γ-connected set satisfying a,b≤ E≤D. We have that $E\not\leq A$, $E\not\leq B$, $E'vAvB=\underline{1}$, $E\wedge A\wedge B=\underline{0}$. This shows that E is not γ– connected in (X,ι) a contradiction.

Definition 3.11: Let (X, T) be an L – topological space, D ε L^x and F denote the set of all γ closed sets in (X, T). A mapping P: M*(D) → F is called an γ- remote neighborhood mapping on Dk, if for each e ε M*(D), it holds that e $\leq$ p(E).

Example 3.12: Let $X_1 \cap X_2 = \emptyset$,X= $X_1 \cup X_2$, $L = [0, 1]$.Defined fuzzy set (C_a,C_b) ε $[0,1]^x$ as follows.

$$(C_a,C_b)\,(x) = \begin{cases} a & if x \ \varepsilon X_1 \\ b & if x \ \varepsilon X_2 \end{cases}$$

Let δ = {Ø , $\underline{1}$, ($C_{0.5}$, $C_{0.5}$)}. Then δ is a [0, 1] – topology on X. $\forall e \ \varepsilon$ M(L^X), define

$$P(e) = \begin{cases} (C_{0.5}, C_{0.5}) & \text{if } e \underline{\not\leq} (C_{0.5}, C_{0.5}); \\ 0 & \text{if } e \underline{\not\leq} (C_{0.5}, C_{0.5}); \end{cases}$$

Then P is a γ remote neighborhood mapping.

Theorem 3.13: Let (X, T) be an L – topological space, D ε L^x. Then D is γ connected if, and only if for any two coprime elements a, b ε M* (D) and any γ - remote neighborhood mapping P: M*(D)$\rightarrow$F, there exist finite many coprime elements e_1 = a, e_2, …, e_n = b in D such that D' V P (e_i) V P (e_{i+1}) ≠ $\underline{1}$, I = 1, 2, …, n – 1.

Proof: Suppose that D is not γ connected. Then there exist two γ- closed sets A, B such that D$\underline{\not\leq}$ A, D$\underline{\not\leq}$ B, D'$\vee$ A$\vee$ B = $\underline{1}$, D$\wedge$ A $\wedge$B = $\underline{0}$. Define a pre-remote neighborhood mapping P: M*(D) $\rightarrow$F, such that $\forall e \ \varepsilon$ M*(D),

$$P(e) = \begin{cases} A & \text{if } e \underline{\not\leq} B; \\ B & \text{if } e \underline{\not\leq} B. \end{cases}$$

Take a,b∈M*(D) such that a$\underline{\not\leq}$A and b$\underline{\not\leq}$B. Then for arbitrary flite many coprime elements e_1= α,e_2,…..,e_n= b in D, there exists an i such that D'vP(e_i)v D'vP(e_{i+1}) =$\underline{1}$, this contradicts the condition of Theorem. Thus the sufficiency is proved. Conversely, suppose that there exist two coprime elements a,b∈M*(D) and an γ-remote neighborhood mapping P:M*(D)→F such that for arbitrary finite many coprime elements e_1=α,e_2,. . . ,e_n=b in D, the following fact is not true: D'vP(e_i)v D'vP(e_{i+1}) ≠$\underline{1}$, i=1,2….n-1. In this case, we say that α and b cannot be linked. Let A={e∈M*(D):a and e can be linked}, B={e∈M*(D)|a and e cannot be linked}. Then∀c∈A and ∀d∈B, we have that D'vP$_{(c)}$v D'vP$_{(d)}$ =1. Put A=∧{P(c)|c∈A }∧ P(d)|d B}. Obviously we have that D A, D B, D'vAvB=$\underline{1}$, D'VAVB=$\underline{0}$. This shows that D is not γ–connected.

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