



CERTAIN STRUCTURES OF DOUBLE FUZZY SOFT IDEALS

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Abstract:

In this article, we introduce the notion of double fuzzy soft sub ideal and double fuzzy soft ideal with examples. We have proved some interesting results which are very close to the results which are very closer to the results fuzzy soft ideal in BCK-Algebras.

Key Words: Soft Set, Fuzzy Soft Set, Double Fuzzy Set, Ideal, Left Ideal, Inclusion & BCK-Algebra

Introduction:

Molodtsov [1999] introduced the concept of soft sets that can be seen as a new mathematical theory for dealing with uncertainty. Molodtsov applied this theory to several directions, and then formulated the notions of soft number, soft derivative, soft integral, etc. The soft set theory has been applied to many different fields with great success. Maji et al. [2002] worked on theoretical study of soft sets in detail, and [2003] presented an application of soft set in the decision making problem using the reduction of rough sets. Chen et al. proposed parametrization reduction of soft sets, and then Kong et al. presented the normal parametrization reduction of soft sets. Maji et al. [2003] presented the concept of the fuzzy soft sets (fs-sets) by embedding the ideas of fuzzy sets [1965]. By using this definition of fs-sets many interesting applications of soft set theory have been expanded by some researchers. Roy and Maji gave some applications of fs-sets. Som defined soft relation and fuzzy soft relation on the theory of soft sets. Aktas. and Cagman and Enginoglu [2010] redefined operations of the soft sets which are more functional for improving several new results. By using these new operations, Cagman and Enginoglu [2010] presented a soft matrix theory. Cagman et al. [2010] defined a fuzzy parameterized soft set theory and its decision making method. In this work, we redefine the fs-sets and their operations, and then define fuzzy soft aggregation operator which allows constructing more efficient decision processes. We finally give an example which shows that the method can be successfully applied to many problems containing uncertainties.

2. Preliminaries:

Throughout the paper, U refers to an initial universe, E is a set of parameters and $P(U)$ is the power set of U . $\subset$ and $\supset$ stand for proper subset and super set, respectively. In this section, we cite the fundamental definitions that will be used in the sequel;

Definition 2.1: An Algebraic system $(X, *, 0)$ of type $(1, 2)$ is called a Double BCK-algebra if it satisfies the following conditions;

- (DBCK-1) $((x^2 * y^2) * (x^2 * z^2) * (z^2 * y^2)) = 0$
- (DBCK-2) $((x^2 * (x^2 * y^2)) * y^2 = 0$
- (DBCK-3) $x^2 * x^2 = 0$
- (DBCK-4) $0^2 * x^2 = 0$
- (DBCK-5) $x^2 * y^2 = 0$ and $y^2 * x^2 = 0 \Rightarrow x = y$, for all $x, y, z, \in X$.

Definition 2.2: A Double fuzzy soft subset δ of X is called a Double fuzzy left soft ideal if

- (DFI-1) $\delta^2(0) \geq \delta^2(x)$
- (DFI-2) $\delta^2(x) \geq \inf\{\delta^2(x * y), \delta^2(y)\}$
- (DFI-3) $\delta^2(xa) \geq \inf\{\delta^2(x * y), \delta^2(a)\}$, for all $x, y, a \in X$.

A Double fuzzy soft subset δ is called a double fuzzy right soft ideal if it satisfies (DF-1), (DF-2) and (DF-4) $\delta^2(ax) \geq \inf\{\delta^2(x * y), \delta^2(a)\}$, for all $x, y, a \in X$.

A doublefuzzy soft subset δ of X is called a double fuzzy soft ideal if it is both left and right double fuzzy soft ideal of X .

Definition 2.3: A double fuzzy soft subset δ of X is called a double left fuzzy soft -r-ideal if

- (DLFr-1) $\delta^2(0) \geq \delta^2(x)$
- (DLFr-2) $\delta^2(x * z) \geq \inf\{\delta^2(x * y) * z, \delta^2(y * z)\}$
- (DLFr-3) $\delta^2(xa) \geq \inf\{\delta^2(x), \delta^2(a)\}$, for all $x, y, z, a \in X$.

Example 2.4: Let $X = \{0, 1, 2\}$ be semi group with following cayley tables;

*	0	1	2
0	0	0	0
1	2	0	1
2	2	1	0

◦	0	1	2
0	0	0	0
1	0	1	0
2	0	0	2

Define a double fuzzy soft sub set $\delta^2 : X \rightarrow [0,1]$ by $\delta^2(0) = 0.4$ and $\delta^2(x) = 0.5$, for all $x \neq 0$. Then by usual calculations, we can prove that δ is a doublefuzzy soft left -r-ideal of X .

Definition 2.5: A soft set f_A over U is defined as $f_A: E \rightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. In other words, a soft set U is a parameterized family of subsets of the universe U . For all $e \in A$ $f_A(e)$ may be considered as the set of e -approximate elements of the soft set f_A . A soft set f_A over U can be presented by the set of ordered pairs:

$$f_A = \{(x, f_A(x)) / x \in E, f_A(x) \subseteq P(U)\} \dots \dots \dots (1)$$

Clearly, a soft set is not a set. For illustration, Molodtsov consider several examples in (1999).

If f_A is a soft set over U , then the image of f_A is defined by $\text{Im}(f_A) = \{f_A(a) / a \in A\}$. The set of all soft sets over U will be denoted by $S(U)$. Some of the operations of soft sets are listed as follows.

Definition 2.6: Let $f_A, f_B \in S(U)$. If $f_A(x) \subseteq f_B(x)$, for all $x \in E$, then f_A is called a soft subset of f_B and denoted by $f_A \subseteq f_B$. f_A and f_B are called soft equal, denoted by $f_A = f_B$ if and only if $f_A \subseteq f_B$ and $f_B \subseteq f_A$.

Definition 2.7: Let $f_A, f_B \in S(U)$ and χ be a function from A to B . Then the soft anti-image of f_A under χ denoted by $\chi(f_A)$, is a soft set over U defined by,

$$\chi_{f_A}(b) = \begin{cases} \cap \{f_A(a) / a \in A, \chi(a) = b\}, & \text{if } \chi^{-1}(b) \neq \emptyset \\ 0, & \text{otherwise} \end{cases} \dots \dots \dots (2)$$

for all $b \in B$. And the soft preimage of f_B under χ , denoted by $\chi^{-1}(f_B)$, is a soft set over U defined by $\chi^{-1}_{f_B}(a) = f_B(\chi(a))$, for all $a \in A$.

Note that the concept of level sets in the fuzzy set theory, Cagman et. al [2007] initiated the concept of lower inclusions soft sets which serves as a bridge between soft sets and crisp sets.

Example 2.8: Assume that $U=Z$ is the universal set and $G=Z_b$ is the subset of parameters. We define a soft set f_G by $f_G(0) = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$$f_G(1) = \{0, 2, 4, 6, 8, 10\}, f_G(2) = \{1, 3, 4, 6, 7\}, f_G(3) = \{0, 2, 3, 6, 9\}, f_G(4) = \{1, 3, 4, 6, 7\}$$

$$f_G(5) = \{0, 2, 4, 6, 8, 10\}. \text{It is clear that } f_G(1+z) \not\subseteq f_G(1) \cap f_G(z), \text{ implying } f_G \text{ is not a soft int-group over } U.$$

Definition 2.9: For any subset A of E , a soft set λ_A over U is a set, defined by a function λ_A , representing the mapping $\lambda_A: E \rightarrow P(U)$. A soft set over U can also be represented by the set of ordered pairs $\lambda_A = \{(x, \lambda_A(x)) ; x \in E, \lambda_A(x) \in P(U)\}$. Note that the set of all soft sets over U will be denoted by $S(U)$.

Definition 2.10: Let $\lambda, \mu \in S(U)$. Then

- ✓ If $\lambda(e) = \emptyset$ for all $e \in E$, λ is said to be a null soft set, denoted by $\emptyset$.
- ✓ If $\lambda(e) = U$ for all $e \in E$, λ is said to be an absolute soft set, denoted by U .
- ✓ λ is a soft subset of μ , denoted $\lambda \subseteq \mu$, if $\lambda(e) \subseteq \mu(e)$ for all $e \in E$.
- ✓ Soft union of λ and μ , denoted by $\lambda \cup \mu$, is a soft set over U and defined by $\lambda \cup \mu: E \rightarrow P(U)$ such that $(\lambda \cup \mu)(e) = \lambda(e) \cup \mu(e)$ for all $e \in E$.
- ✓ $\lambda = \mu$, if $\lambda \subseteq \mu$ and $\lambda \supseteq \mu$.
- ✓ Soft intersection of λ and μ , denoted by $\lambda \cap \mu$, is a soft set over U and defined by $\lambda \cap \mu: E \rightarrow P(U)$ such that $(\lambda \cap \mu)(e) = \lambda(e) \cap \mu(e)$ for all $e \in E$.
- ✓ Soft complement of λ is denoted by λ^c and defined by $\lambda^c: E \rightarrow P(U)$ such that $\lambda^c(e) = U/\lambda(e)$ for all $e \in E$.

Definition 2.11: Let E be a parameter set, $S \subset E$ and $\lambda: S \rightarrow E$ be an injection function. Then $S \cup \lambda(s)$ is called extended parameter set of S and denoted by ξ_S .

If $S=E$, then extended parameter set of S will be denoted by ξ .

3. Properties of Double Fuzzy Soft Ideals:

Theorem 3.1: Every double fuzzy left (resp., right) soft r-ideal of X is a double fuzzy left (resp., right) soft ideal of X .

Proof: Let δ be a doublefuzzy left soft r-ideal of X . Then δ satisfies the conditions of (DF-1) and (DFr-2) of definition 3.2.4.

$$\text{We have } \delta^2(x*z) \geq \inf\{\delta^2(x*y)*z, \delta^2(y*z)\}$$

Putting $z=0$, we get

$$\delta^2(x*0) \geq \inf\{\delta^2(x*y)*0, \delta^2(y*0)\}.$$

Thus $\delta^2(x*) \geq \inf\{\delta^2(x*y), \delta^2(y)\}$. Hence δ is a double fuzzy left softideal of X .

Example 3.2: Let $X = \{p, q, r\}$ be semi group with following cayley tables;

*	p	q	r
p	p	p	p
q	r	p	q
r	r	q	p

$\circ$	p	q	r
p	p	p	p
q	p	q	p
r	p	p	r

Define a double fuzzy soft subset $\delta^2 : \rightarrow [0,1]$ by $\delta^2(0) = 0.7$ and $\delta^2(x) = 0.4$, for all $x \neq 0$. Then by usual calculations, we can prove that δ is a double fuzzyleft soft ideal of X . But it is not doublefuzzy left soft r -ideal of X . Since $\delta^2(q * p) \leq \inf \{ \delta^2(q * p) * p, \delta^2(p * p) \}$.

Theorem 3.3: Let δ be a double fuzzy left (resp., right) softideal of X . Then the non-empty level set δ^β is also a double fuzzy left (resp., right) soft ideal of X .

Proof: We have δ is a doublefuzzy left soft ideal of X .

If $x, y, a \in \delta^\beta$, then $\delta^2(x) \geq \beta$, $\delta^2(y) \geq \beta$ and $\delta^2(a) \geq \beta$.

- ✓ We have $\delta^2(0) \geq \delta^2(x) \geq \beta$, thus $\delta^2(0) \geq \delta^\beta(x)$.
 - ✓ Define $\beta = \inf \{ \delta^2(x * y), \delta^2(y) \}$. We have $\delta^2(x) \geq \beta$, then $\delta^2(x) \geq \beta = \inf \{ \delta^2(x * y), \delta^2(y) \}$.
 - ✓ Define $\beta = \inf \{ \delta^2(x), \delta^2(a) \}$. We have have $\delta^2(xa) \geq \beta$, then $\delta^2(xa) \geq \beta = \inf \{ \delta^2(x * y), \delta^2(a) \}$.
- Hence δ^β is a doublefuzzy left soft ideal of X .

Definition 3.4: Let λ and δ be the doublefuzzy soft subsets in a set X . The Cartesian product $\lambda \times \delta : X \times X \rightarrow [0,1]$ is defined by $(\lambda \times \delta^2)(x, y) = \inf \{ \lambda^2(x), \delta^2(y) \}$ for all $x, y \in X$.

Theorem 3.5: Let λ and δ be a double fuzzy left (respectively right) soft ideal of X , then $\lambda \times \delta$ is also double fuzzy left (respectively right) soft ideal of X .

Proof: For any $(x, y) \in X \times X$. We have

- ✓ $(\lambda^2 \times \delta^2)(0, 0) = \inf \{ \lambda^2(0), \delta^2(0) \} \geq \inf \{ \lambda^2(x), \delta^2(y) \} = (\lambda^2 \times \delta^2)(x, y)$
- ✓ For any $(x_1, x_2), (y_1, y_2) \in X \times X$, We have
 $(\lambda^2 \times \delta^2)(x_1, x_2) = \inf \{ \lambda^2(x_1), \delta^2(x_2) \} \geq \inf \{ \inf \{ \lambda^2(x_1 * y_1), \delta^2(y_1) \}, \inf \{ \delta^2(x_2 * y_2), \delta^2(y_2) \} \}$
 $= \inf \{ \inf \{ \lambda^2(x_1 * y_1), \delta^2(x_2 * y_2) \}, \inf \{ \delta^2(y_1), \delta^2(y_2) \} \} = \inf \{ (\lambda^2 \times \delta^2)(x_1, x_2) * (y_1, y_2), (\lambda^2 \times \delta^2)(y_1, y_2) \}$
- ✓ For any $x, a \in X$ then $xa \in X$ and $(x, y), (a_1, a_2) \in X \times X$, We have
 $(\lambda^2 \times \delta^2)(x_1, y_1)(a_1, a_2) = (\lambda^2 \times \delta^2)(xa_1, xa_2) = \min \{ \lambda^2(xa_1), \delta^2(ya_2) \}$
 $\geq \inf \{ \inf \{ \lambda^2(x), \lambda^2(a_1) \}, \inf \{ \delta^2(y), \delta^2(a_2) \} \} \geq \inf \{ \inf \{ \lambda^2(x), \lambda^2(y) \}, \inf \{ \delta^2(a_1), \delta^2(a_2) \} \}$
 $= \inf \{ (\lambda^2 \times \delta^2)(x, y), (\lambda^2 \times \delta^2)(a_1, a_2) \}.$

Theorem 3.6: Let λ and δ be double fuzzy subsets of X such that $(\lambda \times \mu)$ is double left (respectively right) fuzzy soft ideal of $X \times X$. Then

- ✓ either $\lambda^2(0) \geq \lambda^2(x)$ or $\delta^2(0) \geq \delta^2(x)$
- ✓ if $\lambda^2(0) \geq \lambda^2(x)$ then either $\lambda^2(0) \geq \lambda^2(x)$ or $\delta^2(0) \geq \delta^2(x)$
- ✓ if $\delta^2(0) \geq \delta^2(x)$, then either $\lambda^2(0) \geq \lambda^2(x)$ or $\lambda^2(0) \geq \delta^2(x)$ for all $x \in X$.

Proof: By using reduction and absurdity, we can prove the theorem easily.

Theorem 3.7: Let λ and δ be the double fuzzy subset of X . If $\lambda \times \delta$ is a left (respectively right) fuzzy soft ideal of $X \times X$, then either λ or δ is a doublefuzzy left (respectively right) soft ideal of X .

By theorem 3.3.6 of (i), without loss of generality, we assume that $\lambda^2(0) \geq \delta^2(x)$ for all $x \in X$. By theorem 3.3.6 of (iii) that either $\lambda^2(0) \geq \lambda^2(x)$ or $\lambda^2(0) \geq \delta^2(x)$ for all $x \in X$.

$$\text{If } \lambda^2(0) \geq \delta^2(x), \text{ then } (\lambda^2 \times \delta^2)(0, x) = \inf \{ \lambda^2(0), \delta^2(x) \} \text{-----} \textcircled{1}$$

Since $\lambda \times \delta$ is a doublefuzzy soft left ideal of $X \times X$, therefore for all $(x_1, x_2), (y_1, y_2), (a_1, a_2) \in X \times X$, then

$$\begin{aligned} (\lambda^2 \times \delta^2)(x_1, x_2) &\geq \inf \{ (\lambda^2 \times \delta^2)(x_1, x_2) * (y_1, y_2), (\lambda^2 \times \delta^2)(y_1, y_2) \} \text{ and} \\ (\lambda^2 \times \delta^2)((x_1, x_2)(a_1, a_2)) &\geq \inf \{ (\lambda^2 \times \delta^2)(x_1 * a_1), (\lambda^2 \times \delta^2)(a_1, a_2) \} \\ (\lambda^2 \times \delta^2)(x_1, x_2) &\geq \inf \{ (\lambda^2 \times \delta^2)(x_1 * y_1)(x_2 * y_2), (\lambda^2 \times \delta^2)(y_1, y_2) \} \text{ and} \\ (\lambda^2 \times \delta^2)((x_1, x_2)(a_1, a_2)) &\geq \inf \{ (\lambda^2 \times \delta^2)(x_1, x_2), (\lambda^2 \times \delta^2)(a_1, a_2) \} \end{aligned}$$

If $x_1 = y_1 = a_1 = 0$

$$\begin{aligned} (\lambda^2 \times \delta^2)(0, x_2) &\geq \inf \{ (\lambda^2 \times \delta^2)(0 * 0)(x_2 * y_2), (\lambda^2 \times \delta^2)(0, y_2) \} \text{ and} \\ (\lambda^2 \times \delta^2)((0, x_2)(0, a_2)) &\geq \inf \{ (\lambda^2 \times \delta^2)(0, x_2), (\lambda^2 \times \delta^2)(0, a_2) \} \end{aligned}$$

Using $\textcircled{1}$, we get

$$\delta^2(x_2) \geq \inf \{ \delta^2((x_2 * y_2), \delta^2(y_2) \text{ and } \delta^2(x_2 a_2) \geq \inf \{ \delta^2((x_2 * a_2), \delta^2(a_2) \}$$

This completes the proof.

Conclusion:

In this paper, we introduce the notion of double fuzzy soft sub ideal and double fuzzy soft ideal with examples. We have proved some interesting results which are very close to the results which are very closer to the results fuzzy soft ideal in BCK-Algebras.

References:

1. H. Aktaş and N. Çağman, Soft sets and soft group, Information Science (2007), pp. 2726–2735.
2. N. Cagman and S. Enginoglu, Soft set theory and uni-int decision making, European Journal of Operational Research 207 (2010), 848–855.

3. Y. B. Jun and S. Z. Song, Subalgebras and closed ideals of BCH-algebras based on bipolar-valued fuzzy sets, Sci. Math. Jpn., Vol.68, No.2, pp.287-297, 2008.
4. Y.B. Jun, Soft BCK/BCI-algebras, Computer Mathematics with Applications 56 (2008), 1408–1413.
5. D. Molodsov, Soft set theory–First results, Computers & Mathematics with Applications (4/5) (1999), pp. 19–31.
6. P.K. Maji and A.R. Roy, An application of Soft set in decision making problem, Computers & Mathematics with Applications (2002), pp. 1077–1083.
7. P.K. Maji, R. Biswas and A.R. Roy, Soft set theory, Computers & Mathematics with Applications (2003), pp. 555–562.
8. P.K. Maji, R. Biswas and A.R. Roy, Fuzzy soft sets, Journal of Fuzzy Mathematics, (3) (2001), pp. 589–602.
9. A. Rosenfeld, Fuzzy groups, Journal of Mathematical Analysis and Applications, (1971), pp. 512–517.
10. Sezgin and A.O. Atagun, On operations of soft sets, Computers and Mathematics with Applications 61 (2011), 1457–1467.
11. L.A. Zadeh, Fuzzy sets, Information Control 8 (1965), 338–353.