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A STUDY ON DIVISOR GRAPHS

P. Vidhya* & B. Senthilkumar**

* Research Scholar, Department of Mathematics, PRIST University, Vallam, Thanjavur, Tamilnadu

** Assistant Professor, Department of Mathematics, PRIST University, Vallam, Thanjavur, Tamilnadu

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Abstract:

We characterized this graphs G and H for were they Cartesian product G, H always a divisor graph. We show that divisor graphs. That the ways we prove that the cycle permutation graphs of the order at least eight are divisor graphs if then only if they are perfect. Some results concerning amalgamation operations about obtaining new divisor graphs from old ones are given. Analyzing set of graphs as its vertex amalgamations of complete set of graphs, we characterize those block graphs that are divisor graphs.

Key Words: Graph, Vertex, Vertices & Complete Graph.

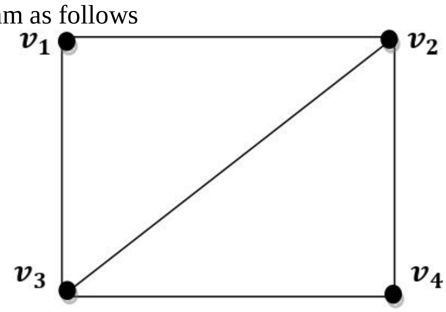
Introduction:

Graphs were among the most ubiquitous models of the both natural and also in human-made structures. They could be used to the many type's models of relations and also process dynamics in physical, biological as well as social systems. Most of the problems of practical interest could be represented by the graphs. For the most part divisor graphs are just mathematical curiosities. Divisor graphs are attractive to mathematicians because they are so easy to describe. In this paper will define divisor graphs in terms of a nonempty finite set of positive integers and basic definitions. The goal of this paper is to explore several families of divisor graphs, as well as looking at divisor graphs and their relationship with their neighborhoods, their complements and products.

Basic Definitions and Examples:

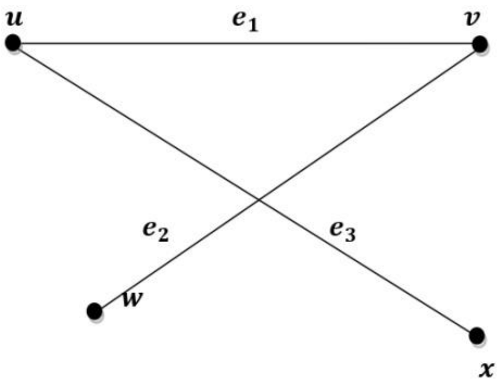
Definition: A graph G consists of a pair $(V(G), E(G))$ where $V(G)$ is a non empty finite set whose elements are called points or vertices or nodes and $E(G)$ is a set of unpaired points of distinct elements of $V(G)$. Always the elements of $E(G)$ were called as lines or edges of graph G.

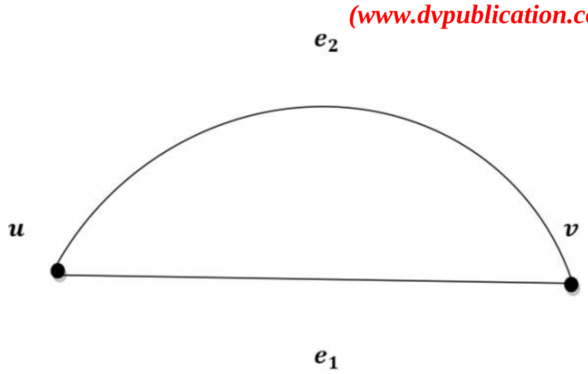
Example: Let G be a $(4,5)$ graph where $V(G) = \{v_1, v_2, v_3, v_4\}$ and $E(G) = \{v_1v_2, v_2v_3, v_3v_4, v_1v_3, v_2v_4\}$. Then G can represented by a diagram as follows



Definition: If $e = \{u, v\} \in E(G)$ the line e is said to join u and v.Then $e = uv$ the points u and v are adjacent. Also say that u and the line e are incident with each other.

Definition: If two distinct lines e_1 and e_2 are incident with a common point then they are called adjacent lines. Here $V(G) = V \; and \; E(G) = E$





e_1 *and* e_2 are adjacent lines

Here e is incident on u and v are incident points.

$$V = \{u,\ v,\ x,\ w\}$$

$$E = \{e_1,\ e_2,\ e_3\} = \{uv, ux, vw\} = \{vu,\ xu,\ wv\}$$

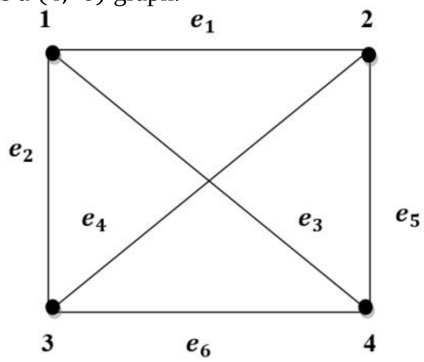
Here,

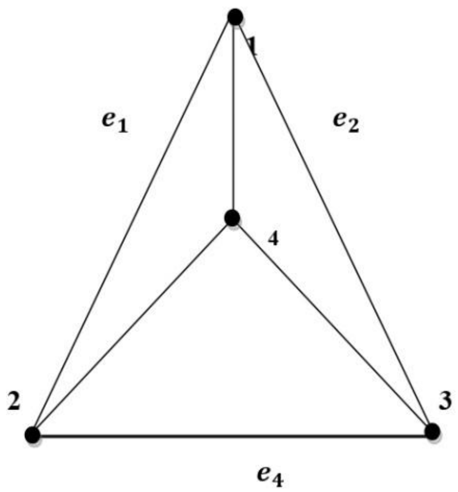
$$e_1 = \{u,\ v\} = \{v,\ u\} = uv = vu.$$

Example:

Let $V = \{1,\ 2,\ 3,\ 4\}$ *and* $E = \{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\}\}$

$G = (V,\ E)$ is a $(4,\ 6)$ graph.

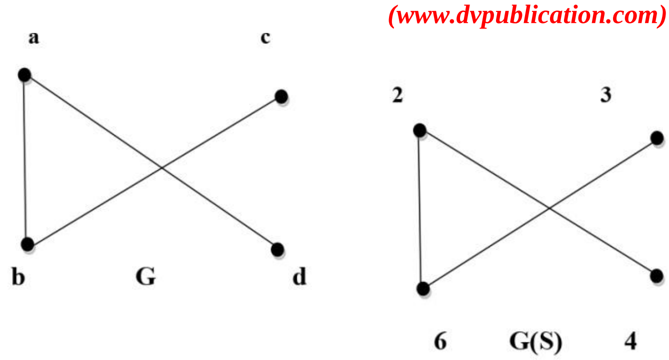




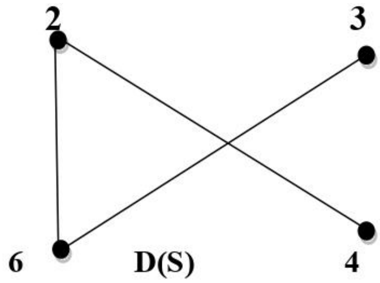
$e_4 = \{2,3\}$ $\quad$ $e_5 = \{2,4\}$ $\quad$ $e_6 = \{3,4\}$

Properties of Divisor Graph:

For the graph G given in figure, the vertices can be labeled with $S = \{2,\ 3,\ 4,\ 6\}$ to produce the divisor graph $G(S)$ and the divisor digraph $D(S)$. The divisor labeling $f: v(G) \rightarrow N$ is defined as
Therefore any divisor graph G has infinitely many divisor labeling.



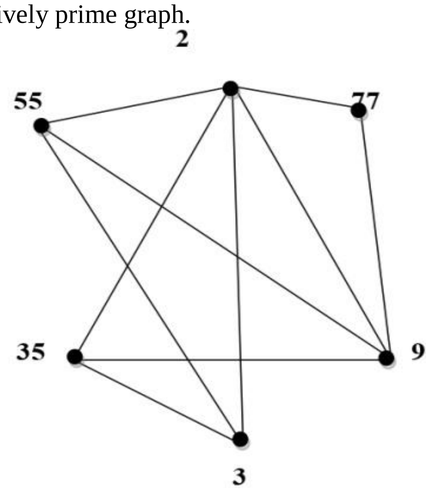
A graph **A divisor labeling of G**



The resulting digraph

The Resulting Digraph:

Let S be a finite non empty set of positive integers. The relatively prime graph RP(S) of S has S as its vertex set and two vertices $i\ and\ j$ are adjacent if $i\ and\ j$ are relatively prime. For graphs F and H we write F = H if F and H are isomorphic. A graph G is a relatively prime graph if G = RP(S) for some finite non empty set S of Positive integers. Hence if G is a relatively prime graph, then there exists a function $f: V(G) \to N$, called a relatively prime labeling of G, such that $G = RP\left(f\left(V(G)\right)\right)$. For $S = \{2, 3, 9, 35, 55, 77\}$, the graph G = RP(S) is shown in figure. Thus G is a relatively prime graph.



A Relatively prime graph G

Definition: Let Z denote the set of all integers. A divisor graph G(S) of a finite subset S of Z in the graph (V, E). Where V = S and $uv \in E$ if and only if either u divides v or v divides u. A graph G is a divisor graph if it is isomorphic to the divisor graph G(S) of some subset S of Z.

Definition: A graph G is called a divisor graph if there exists a labeling f of its vertices with distinct integers such that for any two distinct vertices u and v, uv is an edge of G if and only if $f(u)$ divides $f(v)$ or $f(v)$ divides $f(u)$.

Example: The complete graph K_n.

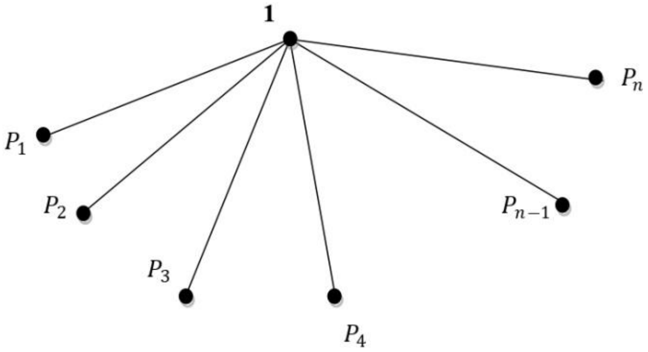
Let a be a non-zero integer. Consider the set $S = \{a, a^2, a^3 \ldots\ldots a^n\}$

Where $n \in N$ (N is the set of positive integers). It follows that $K_n \cong G(S)$

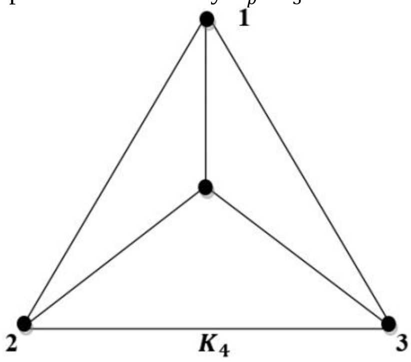
Example: Let us consider the star $K_{1,n}$. Let us take the set $S = \{1, p_1, p_2, \ldots . p_n\}$

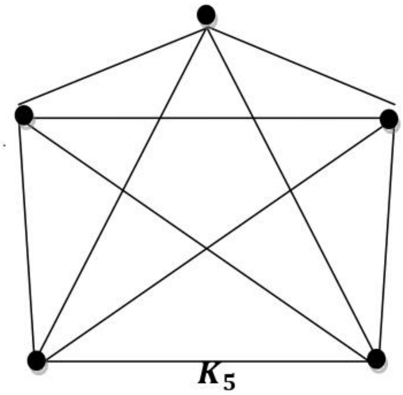
Where p_i stands for the i^{th} prime

Trivially $K_{1,n} \cong G(S)$



Theorem: Every graph is a sub graph of a divisor graph.
Proof: Let a be a non-zero integer. Let us consider the set $S = \{a, a^2, a^3 \ldots \ldots \ldots a^n\}$ where $p \in N$. It immediately shows that $K_p \cong G(S)$.
Since any graph on p vertices is a sub graph of K_p. Therefore every graph is a sub graph of a divisor graph.
Theorem: Let G be a divisor graph with triangles. Then for every vertex v of G, the label $l(v)$ is a common multiple or common divisor of $\{l(x): x \in N(V)\}$ where $l(x)$ denotes the number labeled at x.
Proof: Let u be a vertex of G such that $l(u)$ is a multiples of $l(a)$ and a divisor of $l(b)$. Where $a, b \in N(u)$. Then $l(a)$ divides $l(b)$ and the three vertices u, a, b from a triangle in G.
Complete Graph and Tree Related Divisor Graph
Definition: A graph in any two distinct of points are adjacent is called a complete graph as well as finite graph. In the name of complete graph with p points is denoted by K_p. K_3 is called a triangle





Theorem: Show that if G is complete than $G - v$ is complete
Proof: Let G be a complete graph of order p. To prove that $G - v$ is complete
$G - v$ is a graph obtained by removing the vertex v along with the edges incident with v. If $e = \{u, v\} \in E(G)$ the line e is said to join u and v. Then $e = uv$ the points u and v are adjacent also say that a point u and the line e are incident with each other. Then $G - v$ has $p - 1$ vertices are adjacent to each other. Therefore $G - v$ is complete.
Theorem: If G and H are divisor graph, then $G + H$ is a divisor graph
Proof: Let $V(G) = \{u_1, u_2 \ldots \ldots \ldots \ldots .. u_n\}$ and $V(H) = \{v_1, v_2 \ldots \ldots \ldots \ldots \ldots . v_k\}$
Since G and H are divisor graph, there exist divisor labeling g and h of G and H, respectively.
Let a be the least common multiple of $a_1, a_2 \ldots \ldots \ldots \ldots \ldots a_n$.
Consider the labeling,

Then f is a divisor labeling of $G + H$
Therefore $G + H$ is a divisor graph
Corollary: Every complete multipartite graph is a divisor graph.
Proof: Every bipartite graph is a divisor graph. So $K_{n1,n2}$ is a divisor graph for positive integers n_1 and n_2
If G is an n-partite graph having partite sets $v_1, v_2 \dots \dots \dots \dots \dots . v_n$ such that every vertex of v_i is joined to every vertex of v_j, where $1 \leq i, j \leq n$ and $i \neq j$, then G is called a complete n-partite graph.
If $|v_i| = p_i$ for $i = 1,2 \dots \dots . n$ then we denote G by $K_{p_1,p_2 \dots\dots\dots p_n}$.
These graphs are also called complete multipartite graph.
Thus for $t \geq 3$ and positive integers $n_1, n_2 \dots \dots \dots \dots \dots \dots . . n_t$, the graph $K_{n_1,n_2 \dots\dots\dots n_t} = K_{n_1 n_2} + K_{n_3 n_4} + \cdots \dots \dots \dots + n_t$ the graph divisor graph.
If G and H are divisor graphs, then $G + H$ is a divisor graph.
Hence every multipartite graph is a divisor graph.

Conclusion:

This dissertation is thus a study on divisor graph. This is very useful to Mathematician and very easy to describe. In this paper the several families of divisor graphs and their relationships with their neighborhoods, their complements and products are discussed. Also proved some theorems which are related to divisor graph and prove that every complete graph and tree are also divisor graph.

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