International Journal of Advanced Trends in Engineering and Technology (IJATET)
Impact Factor: 5.965, ISSN (Online): 2456 - 4664
(www.dvpublication.com) Volume 4, Issue 2, 2019

NEW CONCEPT OF FUZZY PLANAR GRAPHS

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Cite This Article: M. Seenu & A. Usha, "New Concept of Fuzzy Planar Graphs", International Journal of Advanced Trends in Engineering and Technology, Volume 4, Issue 2, Page Number 27-32, 2019.

Abstract:

Fuzzy planar graph is a very sensitive and important subclass of fuzzy graph. In this paper, many types of edges are mentioned but especially two types for our fuzzy graphs, namely effective edges and considerable edges. It's Also, a comparative study of Kuratowski's graphs and between of fuzzy planar graph are madden. A very new concept of good effort strong fuzzy planar graph is introduced. Some related results are established. These results always have some applications in subway tunnels, routes, oil/gas pipelines designing's, etc. It is also shown that an image could be represented by a fuzzy planar graph with contraction of such that image can be made with the help of fuzzy planar graph.

Key Words: Fuzzy, Graph, Planar Graph, Edge & Vertex.

Introduction:

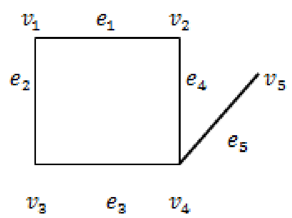
A graph is a convenient way of the representing information to the involving relationship between objects. The objects are represented by vertices and relations by edges. So the graphs are simply models of relations. If there is vagueness in the description of objects or in its relationships or in both, it is natural to assign a "Fuzzy Graph Model". The theory of fuzzy sets was found by Lofti A Zadeh in the year 1965. Zadeh introduced the notion of interval-valued fuzzy sets as an extension of fuzzy sets in which the values of the membership degree are intervals of numbers instead of the numbers. Although the first definition of fuzzy graphs was given by Kaufman, R. T. Yeh and S. V. Bang have also introduced various connectedness concepts in fuzzy graphs during the same time. In 2011, Akram and Dudek defined interval-valued fuzzy graphs and give some operations on them. Rashmanlou and Jun defined complete interval-valued fuzzy graphs. The concept of fuzzy planar graph was introduced by Abdul-Jabbar et al. again; Nirmala and Dhanabal defined special fuzzy planar graphs. Talebi, Rashmanlou and Davvaz investigated some properties of interval-valued fuzzy graphs including regular interval-valued fuzzy graphs.

Basic Definitions:

Basic Concepts in Graph Theory and Fuzzy Graph:

Definition: A graph G(V,E) consists of a set of objects V={v_1,v_2,.....} called the vertices another set E={$e_1, e_2, \ldots\ldots$}whose element are called edges. Such that each e_k is identified with an unordered pair (v_i, v_j) of vertices.

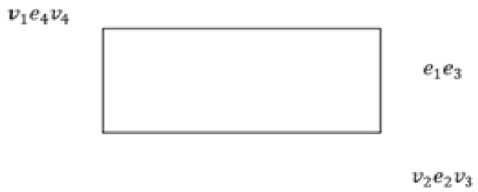
Example:



Here, V={v_1,v_2,.....} and E={$e_1, e_2, \ldots\ldots$}.

Definition: A graph is a finite number of the vertices as well as infinite and finite number of edges are called a finite graph. Otherwise, it is infinite graph.

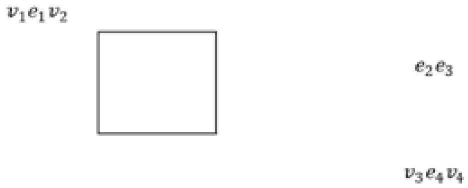
Example:



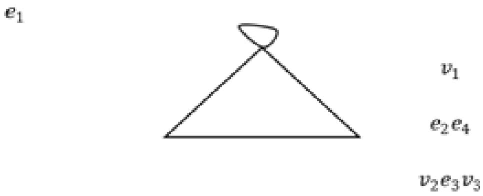
G (V,E) where, V={v_1,v_2, v_3,v_4},E={e_1, e_2, e_3, e_4}.

Definition: A graph G=(V,E) in which all vertices are equal degree is called a regular graph or simply a regular. Otherwise it is called a irregular graph.

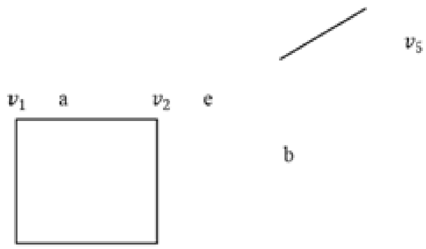
Example:



Where, d(v_1)=2, d(v_2)=2, d(v_3)=2, d(v_4)=2.
Definition: An edge having same vertices as both its end vertices is called self loop or loop.
Example: G:(V,E)



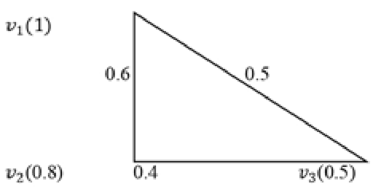
Here, V={v_1, v_2, v_3}, E={e_1, e_2, e_3, e_4}$e_1$is a self loop of G.
Definition: Always A graph G is said to be connected. If there is to be on least one path between every pair of vertices rather than G is disconnected.
Definition: The union of two graphs G_1=(V_1, E_1) and G_2=(V_2, E_2) is another graph $G_1 \cup G_2 = G_3$ such that whose vertex set $V_1 \cup V_2$ and edge $E_1 \cup E_2$.
Definition: The intersection of two graphs $G_1 \cap G_2$ is a graph G_3 consisting only of those vertices and edges that are in both G_1 and G_2.
Definition: A walk is definite as a finite alternative series of vertices and edges beginning and ending with vertices. No edges appear more than one in the walk.
Example: G :(V, E)



Consider the graph G, here the walk is defined is $v_1 a v_2 b v_3 c v_4 d v_1$. There are no edges are repeated. A walk to be begin and end at the same line vertex such as a walk's are called closed walk. Otherwise it is called open walk.
Definition: A graph G is called irregular if there is a vertex which is adjacent to at least one vertex which distinct degree.
Definition: A connected graph G is said to be highly irregular if every vertex of G is adjacent only to vertices with distinct degrees.
Definition: Let X be a universal set and A be a subset of X. then fuzzy set $\tilde{A}$ in X is defined by

$\tilde{A}$ ={(x,$\mu_{\tilde{A}}$(x));xϵX}.where $\mu_{\tilde{A}}$(x) is the membership value of X in $\tilde{A}$.

Example: Consider a finite set A = {a, b, c, d, e, f} and a finite order set N= {0, ½, 1}. Let X be the universal set and A⊆X. $\tilde{A}$ = {(a, 0), (b,1/2), (c,1), (d,1/2), (e,0), (f,1)}.
Example:



G:(σ,μ)

Definition: A fuzzy graph G :(σ,μ) is called strong fuzzy graph if $\mu(u,v)$=$\sigma(u) \wedge \sigma(v)$ for every (u,v) ϵ SXS.
Interval – Valued Fuzzy Graph:
Definition: The interval-valued fuzzy set A in V is defined by A={(x,[μ_A^-(x), μ_A^+(x)]):xϵV}, where μ_A^-(x) and μ_A^+(x) are fuzzy subsets of V such that μ_A^-(x)≤ μ_A^+(x) for all x ϵV.for any two interval-valued sets A=[μ_A^-(x),μ_A^+(x)] and B=[μ_B^-(x),μ_B^+(x)] in we define: A∪B={(x, max(μ_A^-(x),μ_B^-(x)), max(μ_A^+(x), μ_B^+(x)):xϵV},

A∩B={(x, min(μ_A^-(x),μ_B^-(x)), min(μ_A^+(x),μ_B^+(x)):xϵV}. If G^*= (V,E) is a delighted graph, then by an interval self valued fuzzy relation B on a set E we mean an interval-valued fuzzy set. Such thatμ_B^-(xy)≤min(μ_A^-(x),μ_A^-(y)), μ_B^+(xy)≤min(μ_A^+(x), μ_A^+(y)). For every x,y ϵ E.

Operations on Interval-Valued Fuzzy Graphs:

Definition: A graph G^*=(V,E) we mean a pair G=(A,B), where A=[μ_A^-, μ_A^+] is an interval-valued fuzzy set on V and B=[μ_A^-, μ_A^+] is an interval-valued fuzzy relation on E.

Example: Consider a graph G^*= (V,E) such that V={x, y, z}, E={xy, yz, zx}. Let A be an interval-valued fuzzy set of V and let B be an interval-valued fuzzy set of E⊆VXV defined by,A=<(x/0.2, y/0.3, z/0.4), (x/0.4, y/0.5, z/0.5)>, B=<(xy/0.1, yz/0.2, zx/0.1), (xy/0.3, yz/0.4, zx/0.4)>.

Definition: The Cartesian product G_1XG_2 of two interval-valued fuzzy graphs G_1=(A_1, B_1) and G_2=(A_2, B_2) of the graphs G_1^*=(V_1, E_1) and G_2^*=(V_2, E_2) is defined as a pair (A_1XA_2, B_1XB_2)such that

i)$\begin{cases}(\mu_{A1}^- \, X\mu_{A2}^-)(x_1,x_2) = \min\left(\mu_{A1}^-(x_1),\mu_{A2}^-(x_2)\right) \\ (\mu_{A1}^+ \, X\mu_{A2}^+)(x_1,x_2) = \min(\mu_{A1}^+(x_1),\mu_{A2}^+(x_2))\end{cases}$ for all (x_1,x_2)∈V,

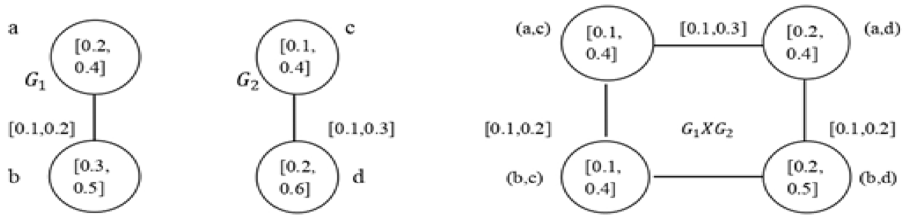
ii)$\begin{cases}(\mu_{B1}^- X\mu_{B2}^-)(x,x_2)(x,y_2) = \min(\mu_{A1}^-(x),\mu_{B2}^-(x_2y_2)) \\ (\mu_{B1}^+ X\mu_{B2}^+)(x,x_2)(x,y_2) = \min(\mu_{A1}^+(x),\mu_{B2}^+(x_2y_2))\end{cases}$ for all x∈ $V_1 and\ (x_2y_2) \in E_2$,

iii)$\begin{cases}(\mu_{B1}^- X\mu_{B2}^-)(x_1,z)(y_1,z) = \min(\mu_{B1}^-(x_1y_1),\mu_{A2}^-(z)) \\ (\mu_{B1}^+ X\mu_{B2}^+)(x_1,z)(y_1,z) = \min(\mu_{B1}^+(x_1y_1),\mu_{A2}^+(z))\end{cases}$ for all z∈ $V_2 and\ (x_1y_1) \in E_1$.

Example: Let G_1^*=(V_1, E_1) and G_2^*=(V_2, E_2) be graphs such thatV_1={a,b}, V_2={c,d},E_1={ab}and E_2={cd}. Consider bi interval valued fuzzy graphs G_1=(A_1, B_1) and $G_2 = (A_2, B_2)$.

Where A_1=<(a/0.2, b/0.3),(a/0.4, b/0.5)>, B_1=<ab/0.1, ab/0.2>,
A_2=<(c/0.1,d/0.2),(c/0.4,d/0.6)>, B_2 =<cd/0.1, cd/0.3>
Then, as it is not difficult to verify

$(\mu_{B1}^- X\mu_{B2}^-)$((a,c)(a,d))=0.1, $(\mu_{B1}^+ X\mu_{B2}^+)$((a,c)(a,d))=0.3,
$(\mu_{B1}^- X\mu_{B2}^-)$((a,c)(b,c))=0.1, $(\mu_{B1}^+ X\mu_{B2}^+)$((a,c)(b,c))=0.2,
$(\mu_{B1}^- X\mu_{B2}^-)$((a,d)(b,d))=0.1, $(\mu_{B1}^+ X\mu_{B2}^+)$((a,d)(b,d))=0.2,
$(\mu_{B1}^- X\mu_{B2}^-)$((b,c)(b,d))=0.1, $(\mu_{B1}^+ X\mu_{B2}^+)$((b,c)(b,d))=0.3.



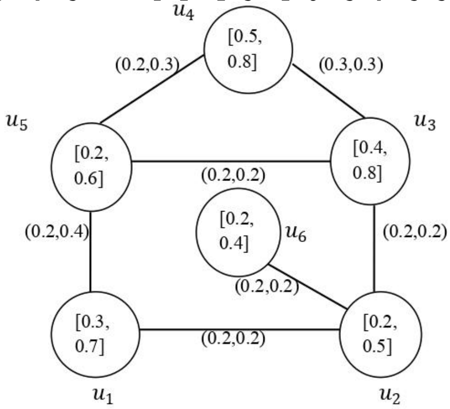
Irregular Interval-Valued Fuzzy Graph:

Definition: Let G = (A,B) be an interval-valued fuzzy graph on G^*. The open degree of a vertex u is defined as deg(u) = (deg^-(u), deg^+(u)) where, deg-(u) = $\sum_{\substack{u\neq v \\ v\in V}} \mu_{B^-(uv)}$ and $deg^+(u)$=$\sum_{u\neq v} \mu_{B^+(uv)}$

Definition: Let G be a connected interval-valued fuzzy graph. If every vertex of G is adjacent to vertices with distinct neighborhood degrees.

Example: Consider an interval-valued fuzzy graph G such that

V={u_1, u_2, u_3, u_4, u_5}E={$u_1u_2, u_2u_3, u_2u_6, u_3u_4, u_3u_5, u_4u_5, u_5u_1$}



By routine computation, we have

deg(u_1) =(0.4,0.6), deg(u_2) =(0.6,0.6), deg(u_3) =(0.7,0.7), deg(u_4) =(0.5,0.6), deg(u_5) =(0.6,0.9), deg(u_6) =(0.2,0.2). Consider a vertex $u_2 \in V$ which is adjacent to the vertices u_1, u_3, u_6 with distinct degrees.

We define $\overline{G}$ = ($\overline{A}, \overline{B}$), G. we need

$\overline{G}$ = ($\overline{A}$, $\overline{B}$) be an interval-valued fuzzy graph, suppose that G = (A,B) is an interval-valued fuzzy graph that satisfies the following conditions:

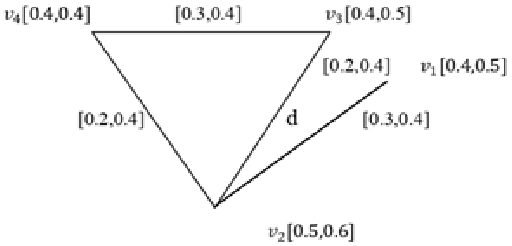
$$\mu(A\hat{}(-((x))\,)\wedge)\ \_(A\hat{}(((y))\,)\,)\ -\ \mu(B\hat{}(-((xy))\,))\le\mu\_(A\hat{}(+\_((x))\,)\,)\wedge\mu\_(A\hat{}(+\_((y))\,)\,)-\mu\_(B\hat{}(+\_((xy))\,)\,)\text{for all } x,y\ \in V$$

Definition: The complement of an interval-valued fuzzy graph G=(A,B) of a graph G^*=(V,E) is an interval-valued fuzzy graph $\overline{G}$ =($\overline{A}$,$\overline{B}$) of $\overline{G^*}$ = (V,VxV) where, $\overline{A}$ = A=[μ_{A^-},μ_{A^+}] and $\overline{B}$=[μ_{B^-},μ_{B^+}] is defined by

μ_{B^-}(x,y) = $\mu_{A^-(x)} \wedge \mu_{A^-(y)}$ - $\mu_{B^-(xy)}$ for all x, y ∈ V.μ_{B^+} $\overline{(x,y)}$ = $\mu_{A^+(x)} \wedge \mu_{A^+(y)}$-$\mu_{B^+(xy)}$for all x,y ∈ V.

Definition: Let G be a connected interval-valued fuzzy graph. Then G is called highly irregular interval-valued fuzzy graph if every vertex of G is adjacent to vertices with distinct degrees.

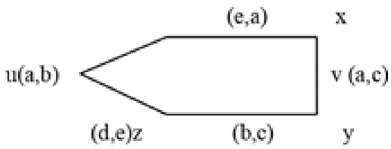
Example: Let G=(A,B) be an interval-valued fuzzy graph. Where A=[μ_{A^-}, μ_{A^+}] and B=[μ_{B^-}, μ_{B^+}]be two interval-valued fuzzy set on a non-empty finite set V and E ⊆ VxV respectively, where V={v_1, v_2, v_3, v_4}.



So we have d(v_1) = [0.3,0.4], d(v_2) = [0.7,1.2], d(v_3) = [0.5,0.8], d(v_4) = [0.5,0.8]. Here the interval-valued fuzzy graph is highly irregular but not neighborly irregular as d(v_3) = d(v_4).

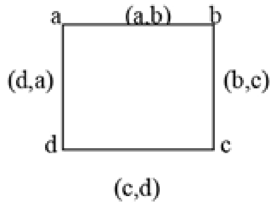
Definition: For two (not necessarily distinct) vertices u and v in a fuzzy graph, A u-v fuzzy walk in fuzzy graph is a sequence of vertices in fuzzy graph, beginning with u and ending at v such that consecutive vertices in w are adjacent in fuzzy graph with $\mu(v_i,\ v_{i+1})$ ≥0 such a fuzzy walk in FG can be expressed as W = $u_0\mu(u_0, v_1)$, $v_1(v_1, v_2)$, $v_{12}(v_2, v_3)$,….., $v_{n-1}(v_{n-1}, v_n)$ where $v_i v_{i+1} \in fuzzy\ graph$ for (0≤I≤n-1). The fuzzy walk W is said to contain each vertex v_i(0≤ $i \le n$) with $\mu(v_i, v_{i+1})$ ≥0 and each edge $v_i v_{i+1}$(0≤i≤n-1).

Example: uμ(a,b)xμ(a,c)y μ(b,c)zμ(d,e)v represents a fuzzy walk in fuzzy graph. The vertices u and v are terminal vertices.



Definition: A fuzzy walk whose initial and terminal vertices are distinct with $\mu(v_i, v_{i+1})$ ≥0 is an open fuzzy walk; otherwise it is closed fuzzy walk with $\mu(v_i, v_{i+1})$ ≥0.

Example:



Here, a μ(a,b) b μ(b,c) c μ(c,d) d μ(d,a) a represents closed fuzzy walk of fuzzy graph. And another, a μ(a,b) b μ(b,c) c μ(c,d) represents open fuzzy walk.

Definition: Let G = (A,B) be a connected fuzzy graph. G is said to be a neighbourly irregular interval-valued fuzzy graph if every two adjacent vertices of G have distinct degree.

Definition: Let G =(A,B) be a connected interval-valued fuzzy graph. G is said to be a strongly irregular interval-valued fuzzy graph if every pair of vertices in G have distinct degrees.

Example:

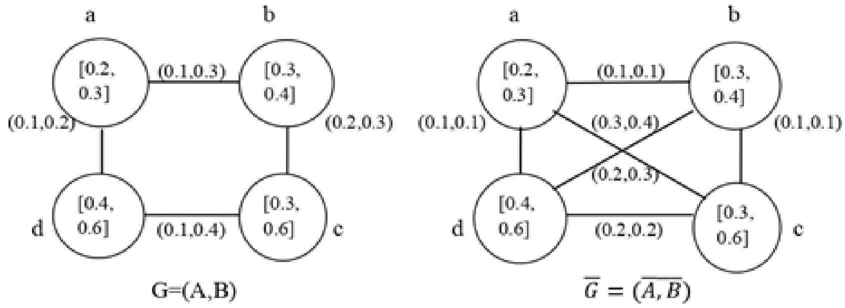
u(0.5,0.6) (0.1,0.2) v(0.3,0.4)

x(0.3,0.4) (0.3,0.4) w(0.2,0.3)

Here d(u) = (0.2,0.4), d(v) = (0.3,0.5), d(w) = (0.5,0.7), d(x) = (0.4,0.6).

Theorem: The complement of highly irregular interval valued sectional fuzzy graph need not be highly irregular.

Proof: To the every vertex and adjacent vertices with the distinct degrees (or) the non adjacent vertices with distinct degrees may happen to be adjacent vertices with same degrees. This contradicts the definition of highly irregular interval-valued fuzzy graph. Consider a graph G and $\overline{G}$. We see that $\overline{G}$ is not highly irregular because the degrees of the adjacent vertices of a and b are the same.



Theorem: Let G be an interval-valued fuzzy graph. Then G is highly irregular interval-valued fuzzy graph and neighborly irregular interval-valued fuzzy graph if and only if the degrees of all vertices of G are distinct.

Proof: Let G = (A,B) be an interval-valued fuzzy graph where A=[μ_{A^-}, μ_{A^+}] and B = [μ_{B^-}, μ_{B^+}] be two interval-valued fuzzy sets on a non-empty finite set V and VxV respectively.

Let V ={v_1, v_2,……,v_n} we assume that G is highly irregular and neighborly irregular interval-valued fuzzy graphs. Let the adjacent vertices of u_1 be u_2, u_3,…..,u_n with degrees [k_2^-, k_2^+], [k_3^-, k_3^+],…….,[k_n^-, k_n^+] respectively. As G is highly and neighbourly irregular, d(u_1)≠ d(u_2)≠d(u_3) ≠,……≠d(u_n).

This means that every two adjacent vertices have distinct degrees and to every vertex the adjacent vertices have distinct degrees.

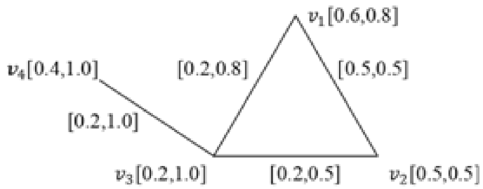
Hence, G is neighbourly irregular and highly irregular interval-valued fuzzy graphs.

Regular Interval-Valued Fuzzy Graph:

Definition: Let G=(V,E,μ,ρ) be an interval-valued fuzzy graph. The order of G, denoted O(G), is defined as O(G) =[O^-(G), O^+(G)], where O^-(G) =$\sum_{v\in V}\mu^-$(v);O^+(G) =$\sum_{v\in V}\mu^+$(v). Similarly, the size of G, denoted S(G), is defined as S(G) =[S^-(G), S^+(G)], where S^-(G) = $\sum_{vw\in E}\rho^-$(vw); S^+(G) = $\sum_{vw\in E}\rho^+$(vw).

Definition: An interval-valued fuzzy graph G =(V,E,μ,ρ) is said to be a strong interval-valued fuzzy graph if ρ^-(vw) = μ^-(v) $\wedge\mu^-$(w) and ρ^+(vw) = μ^+(v) $\wedge\mu^+$(w), for every v,w ∈ E.

Example: Let G =(V,E,μ,ρ) be an interval-valued fuzzy graph,
where V={v_1, v_2, v_3, v_4}, E={$v_1v_2, v_1v_3, v_2v_3, v_3v_4$} with $\mu(v_1)$ =[0.6,0.8], $\mu(v_2)$=[0.5,0.5], $\mu(v_3)$=[0.2,1.0], $\mu(v_4)$=[0.4,1.0];
$\rho(v_1v_2)$ = [0.5,0.5], $\rho(v_1v_3)$=[0.2,0.8], $\rho(v_2v_3)$=[0.2,0.5], $\rho(v_3v_4)$=[0.2,1.0].



In G, We have O(G) = [1.7,3.3] and S(G)=[1.1,2.8].

Interval-Valued Fuzzy Planar Graph:

Definition: Let ξ=(A,B), where A=(V,[σ^-,σ^+]) is an interval-valued fuzzy set on V and B=(VXV,[μ^-,μ^+]) is an interval-valued fuzzy set on VXV, such that μ^-(x,y) ≤min{$\sigma^-(x),\sigma^+(y)$} and μ^+(x,y) ≤min{$\sigma^+(x),\sigma^+(y)$} for all (x,y)∈E here A as the interval-valued fuzzy vertex set of ξ and B as the interval –valued fuzzy edge set of ξ respectively.

Definition: An interval-valued fuzzy graph ξ=(A,B) is said to be complete interval-valued fuzzy graph ifμ^-(x,y)=min{σ^-(a),σ^-(b)} and μ^+(x,y)=min{σ^+(x),σ^+(y)}, for all x,yϵV.

Definition: Let interval-valued fuzzy multigraph ξ=(A,B), B contains two edges ((a,b),[$\mu^-(a,b),\mu^+(a,b)$]) and ((c,d),[$\mu^-(c,d),\mu^+(c,d)$]) which are intersected at a point P. now, we calculate the interval numbers $I_{(a,b)} and\ I_{(c,d)}$ for the respective edges. We define the intersecting number at the point P by

$$I_p = [I_p^-, I_p^+] = [I_{(a,b)}^- + I_{(c,d)}^-/2, I_{(a,b)}^+ + I_{(c,d)}^+/2]$$

Here I_p is an interval number in [0,1]

Definition: Let ξ be an interval-valued fuzzy multigraph and for a certain geometrical representation $P_1, P_2, \ldots\ldots, P_k$ be the points of intersections between the edges. Then ξ is said to be interval-valued fuzzy planar graph with degree of planarity f=[f^-, f^+],

Where f^- =1/1+{I_{p1}^+, I_{p2}^++…….+I_{pk}^+}, and f^+ =1/1+{I_{p1}^-, I_{p2}^-+…….+I_{pk}^-}.

Definition: Strength of interval-valued fuzzy edge (a,b) can be measured by the value $I_{(a,b)}$=(a,b)μ^k/min{σ(a), σ(b)}, if $I_{(a,b)}$ ≥0.5, then the fuzzy edge is called strong otherwise weak.

Definition: An interval-valued fuzzy planar graph ξ is called strong interval-valued fuzzy planar graph if the degree of planarity is greater than or equal to [0.67,0.67].

Definition: Let ξ =(A,B) be an interval-valued fuzzy planar graph. Let 0 < c < 0.5 be rational number. An edge (x,y) is said to considerable edge if [μ^-(x,y) / min{$\sigma^+(x), \sigma^+(y)$}, μ^+(x,y) / min{$\sigma^+(x), \sigma^+(y)$}]≥[c,c]. if an edge is not considerable, it is called non considerable edge.

Definition: Let ξ =(A,B) be a strong interval-valued fuzzy planar graph, whose degree of planarity is [1,1] on V. a face of ξ is a region, bounded by the set of edges E' ⊂ VxV, of a geomentric representation of ξ. The strength of the face is {[min{$I^-_{(x,y)}$}, min{$I^+_{(x,y)}$}] / (x,y) ∈ E' }

Theorem: Let ξbe a complete interval-valued fuzzy graph. The degree of planarity f of ξ is given by f =[f^-, f^+], where f^-=1/1+N_p and 1/1+N_p ≤ f^+ ≤1, where N_p is the number of points of intersection between the edges in ξ.

Proof: Let, ξ=(A,B) be an interval-valued fuzzy graph. For the complete interval-valued fuzzy graph,$\mu^-(x,y) = \min\{\sigma^-(x), \sigma^-(y)\}$ and $\mu^+(x,y) = \min\{\sigma^+(x), \sigma^+(y)\}$ for all x,y∈V. let,$P_1, P_2, \ldots\ldots, P_k$ be the points of intersections between the edges in ξ, k being an integer. For any edge (a,b) in a complete interval-valued fuzzy graph, $I^-_{(a,b)}$=μ^-(a,b)/min{$\sigma^-(a), \sigma^-(b)$}≤1 and $I^+_{(a,b)}$=μ^-(a,b)/min{$\sigma^+(a), \sigma^+(b)$}=1.

Therefore, for the point P_1, the point of intersection between the edges (a,b) and (c,d), I^+_{p1} = 1+1/2=1 and I^-_{p1} ≤1+1/2=1.

Hence, I^+_{p1} =1 and I^-_{p1} ≤ 1 for i=1,2,….,k

Now, f^-=1/1+I^+_{p1}+I^+_{p2}+…….+I^+_{pk}

= 1/1+(1+1+……+1) = 1/1+N_p and it is obvious that,

Where N_p is the number of point of intersection between the edges in ξ.

Therefore, the degree of planarity f is given by f=[f^-,f^+] where,f^-=1/1+N_p and 1/1+N_p ≤ f^+ ≤ 1.

Conclusion:

In this paper, interval-valued fuzzy graph have been discussed. Also the interval-valued fuzzy planar graph with 'degree of planarity' is used to measure the nature of planarity of an interval –valued fuzzy planar graph. Some problems have been solved by degree of planarity. And we define a new term called walk on irregular interval-valued fuzzy graph. This is a very interesting concept of interval-valued fuzzy graph. Some theorems have been proved on irregular interval-valued fuzzy graph and regular interval-valued fuzzy graphs.The results discussed in this dissertation may be used to study about various interval-valued fuzzy graph invariants.

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